

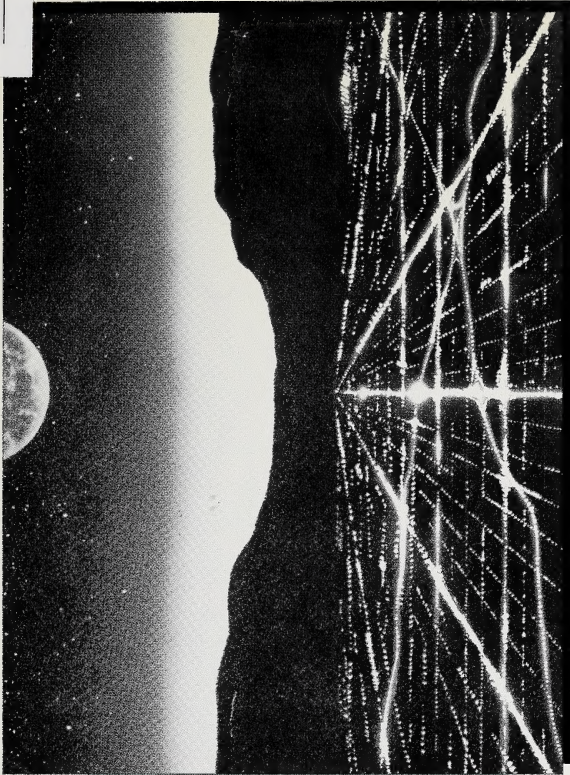
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Mathematics 1

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UNIT 4

Coordinate

Geometry

& Graphing



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W e l c o m e



Distance Learning

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Mathematics 13 Student Module Unit 4 Coordinate Geometry and Graphing Alberta Distance Learning Centre ISBN No. 0-7741-0590-9

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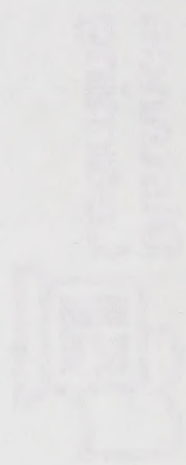
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General Information

This information explains the basic layout of each booklet.

- **What You Already Know and Review** are to help you look back at what you have previously studied. The questions are to jog your memory and to prepare you for the learning that is going to happen in this unit.
- As you begin each **Topic**, spend a little time looking over the components. Doing this will give you a preview of what will be covered in the topic and will set your mind in the direction of learning.
- **Exploring the Topic** includes the objectives, concept development, and activities for each objective. Use your own papers to arrive at the answers in the activities.
- **Extra Help** reviews the topic. If you had any difficulty with **Exploring the Topic**, you may find this part helpful.
- **Extensions** gives you the opportunity to take the topic one step further.
- To summarize what you have learned, and to find instructions on doing the unit assignment, turn to the **Unit Summary** at the end of the unit.
- The **Appendices** include the solutions to **Activities (Appendix A)** and any other charts, tables, etc. which may be referred to in the topics (**Appendix B**, etc.).

Visual Cues

Visual cues are pictures that are used to identify important areas of the material. They are found throughout the booklet.

An explanation of what they mean is written beside each visual cue.



What You Already Know

- reviewing what you already know



Key Idea

- flagging important ideas



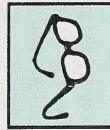
Audiotape

- learning by listening to an audiotape



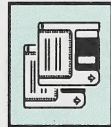
Review

- studying previous concepts



Another View

- exploring different perspectives



Computer Software

- learning by using computer software



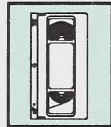
Introduction

- introducing the unit



Solutions

- correcting the activities



Videotape

- learning by viewing a videotape



What Lies Ahead

- previewing the unit



Extra Help

- providing additional study



Print Pathway

- choosing a print alternative



Exploring the Topic

- actively learning new concepts



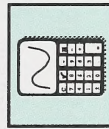
Extensions

- going on with the topic



Calculator

- using your calculator



Graphing Calculator

- using graphing calculator



What You Have Learned

- summarizing what you have learned

Mathematics 13

Course Overview

Mathematics 13 contains 6 units. Beside each unit is a percentage that indicates what the unit is worth in relation to the rest of the course. The units and their percentages are listed below. You will be studying the unit that is shaded.

Unit 1 Number Systems	10%
Unit 2 Polynomials and Factoring	28%
Unit 3 Geometry	18%
Unit 4 Coordinate Geometry and Graphing	18%
Unit 5 Relations	12%
Unit 6 Statistics	14%
	100%

Unit Assessment

After completing the unit, you will be given a mark based totally on a unit assignment. This assignment will be found in the Assignment Booklet.

Unit Assignment - 100%

If you are working on a CML terminal, your teacher will determine what this assessment will be. It may be

Unit Assignment - 50%
Supervised Unit Test - 50%

Introduction to Coordinate Geometry and Graphing

This unit covers topics dealing with coordinate geometry and graphing. Each topic contains explanations, examples, and activities to assist you in understanding coordinate geometry and graphing. If you find you are having difficulty with the explanations and the way the material is presented, there is a section called **Extra Help**. If you would like to extend your knowledge of the topic, there is a section called **Extensions**.

You can evaluate your understanding of each topic by working through the activities. Answers are found in the solutions in the **Appendix**. In several cases there is more than one way to do the question.

Unit 4 Coordinate Geometry and Graphing

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	• What Lies Ahead	
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	• What Lies Ahead	
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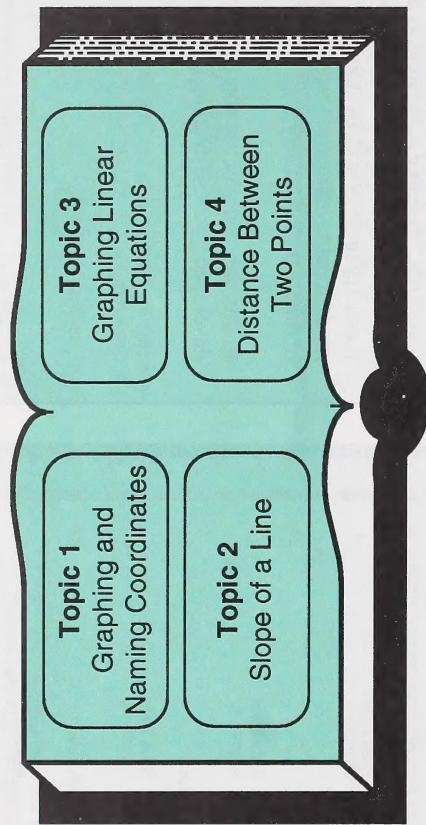
Coordinate Geometry and Graphing

René Descartes was born in 1596 in Tours, France. He led a varied life, spending some time as a soldier and some time as a mathematician. Descartes published the first paper on analytic geometry in 1619 and, by doing so, brought about the birth of modern mathematics. Others, like Fermat, continued with work in the field of geometry.

In this unit you will use some of the ideas and the procedures developed by these mathematicians.

Unit 4

Coordinate Geometry and Graphing

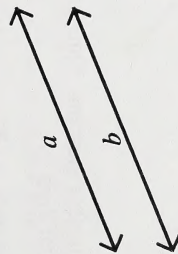




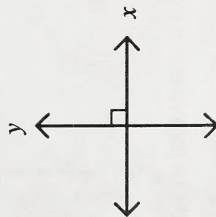
What You Already Know

Do you remember the following?

- **Parallel lines** have no common points no matter how far they are extended. In the diagram, line $a \parallel$ line b . The symbol $\parallel$ means parallel.



- Lines which intersect at right angles are called **perpendicular lines**. In the diagram, line $x \perp$ line y . The symbol $\perp$ means perpendicular.



- The **Pythagorean theorem** states that in any right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

In triangle ABC the hypotenuse is labelled c . The hypotenuse is the longest side of $\triangle ABC$. The hypotenuse is opposite the right angle.

In short, the Pythagorean theorem can be stated as $c^2 = a^2 + b^2$.

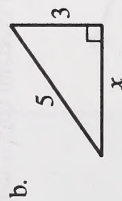
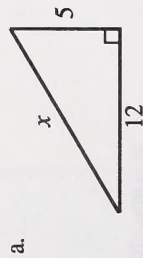




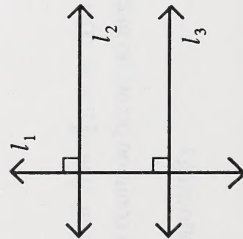
Review

Try the following review questions.

1. What are the lengths of the unknown sides in the following two triangles?

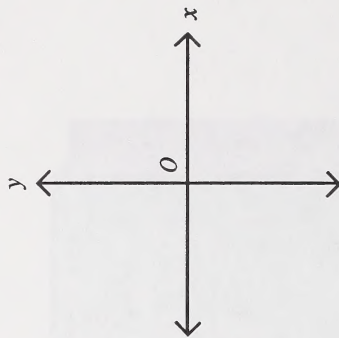


2. Look at the following diagram. Name the parallel lines. Name the perpendicular lines too.



3. Suppose the following sketch is a partial coordinate plane.

- a. What would you call the vertical line?
b. What would you call the horizontal line?
c. What would you call the point O where the two lines cross each other?



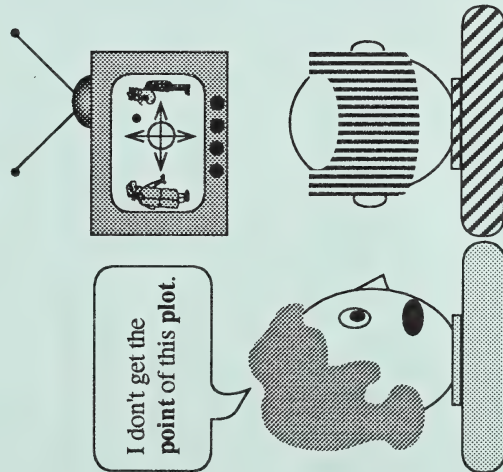
Now go to the **Review** solutions in the **Appendix**.

Topic 1 Graphing and Naming Coordinates



Introduction

In this topic you will be given a chance to tone up your graphing muscles. You are going to review some of the skills you need to plot points in the Cartesian plane.



What Lies Ahead

Throughout the topic you will learn to

1. graph and name coordinates

Now that you know what to expect, turn the page to begin your study of graphing and naming coordinates.



Exploring Topic 1

Activity 1



Graph and name coordinates.

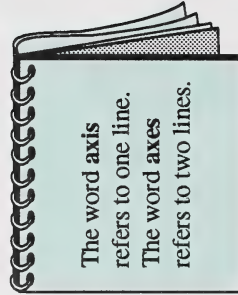
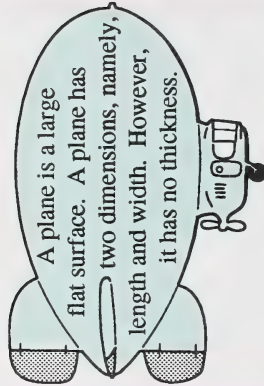
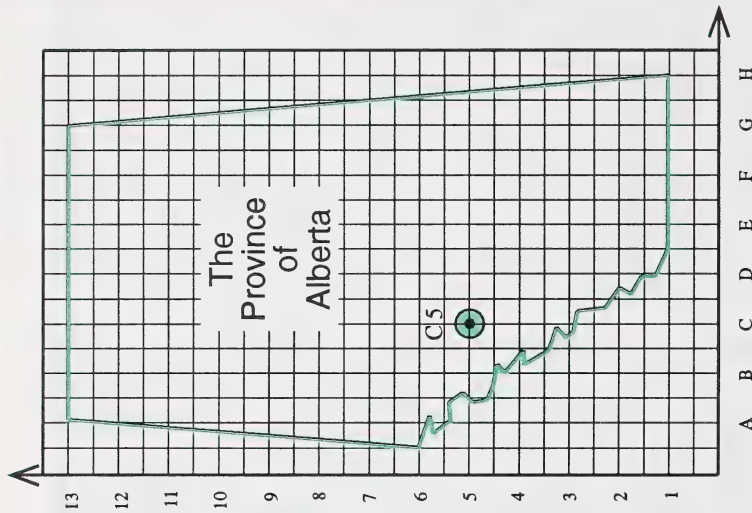
If you want to find a place on a local, provincial, or county map, you need two pieces of information. These are the vertical and the horizontal components.

On the grid shown, the numbers represent the horizontal lines and the letters represent the vertical lines.

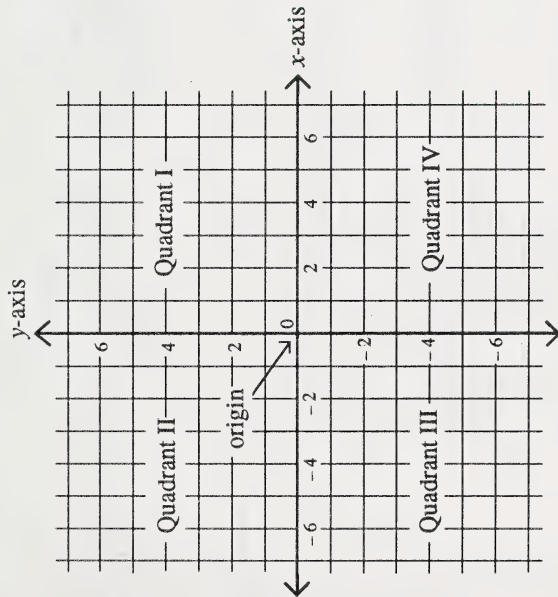
The locator C5, for example, indicates a region on the map. Any place in and around this region is given by the locator C5.

In mathematics you use a similar method for plotting points in the plane.

The **Cartesian plane** or **coordinate plane** is established by first drawing two intersecting lines called the **axes**. These axes are drawn at right angles to each other. Other lines equal intervals apart are drawn parallel to these axes.



The horizontal axis is called the **x-axis**. The vertical axis is called the **y-axis**. The two axes intersect at a point called the **origin**. The four regions formed by the axes are called **quadrants**. The quadrants are numbered **counterclockwise** starting at the upper right. Roman numerals I, II, III, and IV are used to name the quadrants.



Note that the x-axis and the y-axis are numbered outward, upward, and downward from the origin. The numbers on the axes indicate the number of square intervals from the origin.

Using the following coordinate plane, go up two intervals along the y-axis and stop at a horizontal line parallel to the x-axis. This line is called $y = 2$. If you go down the y-axis to -3 , this horizontal line is called $y = -3$.

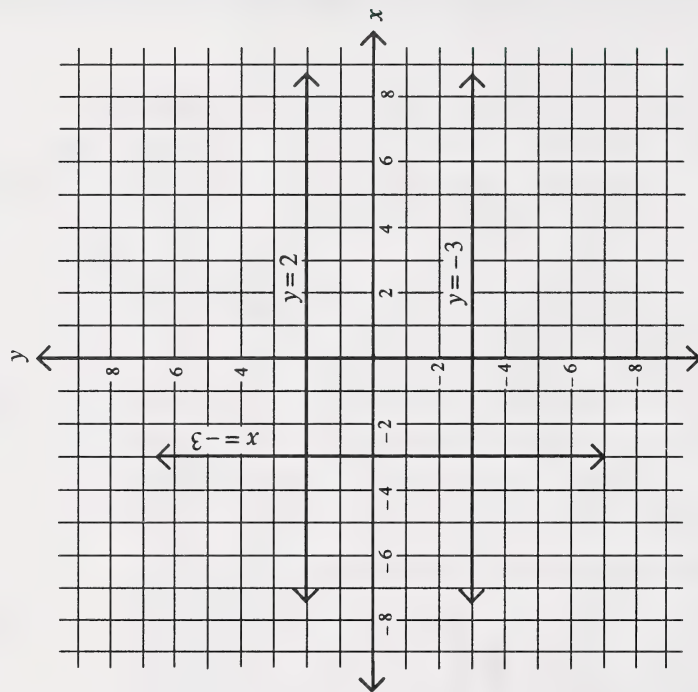
In a coordinate plane the axes are the two main lines which intersect at 90° angles. Arrows are placed at both ends of each axis.



Each quadrant is one quarter or one of four parts of a coordinate plane. The prefix **quad** always indicates four as in the following:

- A quadrilateral is a four-sided polygon.
- Quadraphonic means having four stereo speakers.
- Quadruple means four times as much.

The same is true for lines crossing the x -axis which are parallel to the y -axis. If you go three intervals to the left of the origin, this vertical line is called $x = -3$.



In order to plot points on a coordinate plane, you use the coordinates of the points. **Ordered pairs** name the coordinates of the points. Points may also be identified by using letters of the alphabet.

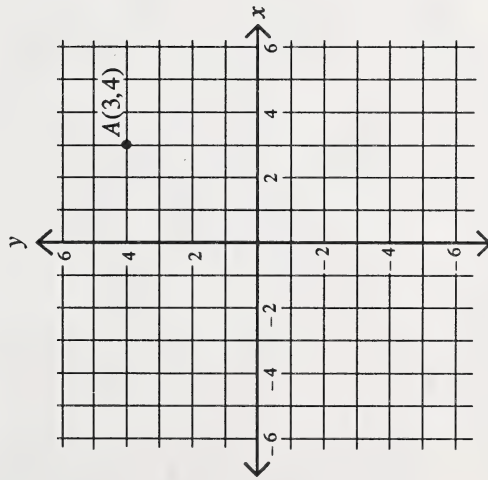
Consider the point $A(3, 4)$ in the coordinate plane.

This is the letter used to name the point on the Cartesian plane.

$A(3, 4)$

The first coordinate gives the position with respect to the x -axis. The point A is three units to the right of the origin.

The second coordinate gives the position with respect to the y -axis. The point A is four units up from the origin.

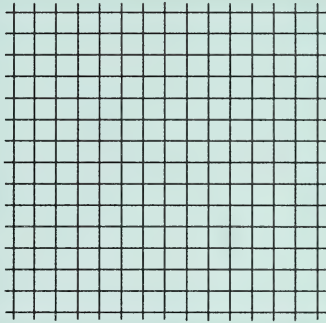


You can make several observations about this point.

- The point is in the first quadrant.
- The line $y = 4$ passes through this point.
- The line $x = 3$ passes through this point.
- The x -coordinate and the y -coordinate are both positive.



- Remember that $(3, 4)$ is read as the coordinate $(3, 4)$.
- Remember that $(3, 4)$ can also be read as the ordered pair $(3, 4)$.
- The coordinate of a point is always placed inside parentheses.
- If the axes in a coordinate plane are removed, the coordinate plane becomes a grid as shown.



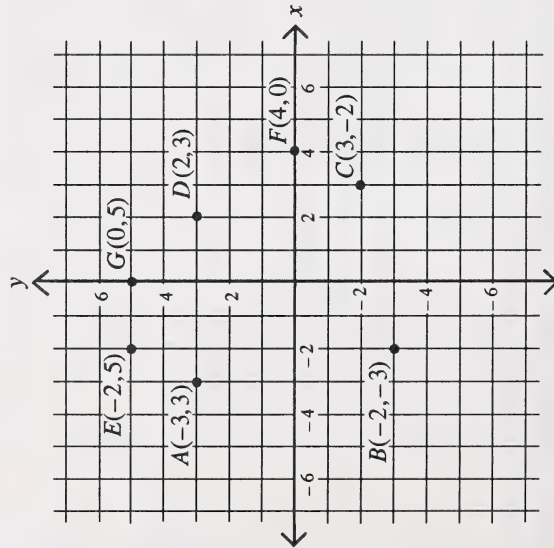
Example 1

Plot the following points in the Cartesian plane.

$A(-3, 3)$, $B(-2, -3)$, $C(3, -2)$, $D(2, 3)$, $E(-2, 5)$

Solution:

Draw a coordinate plane and plot the points.

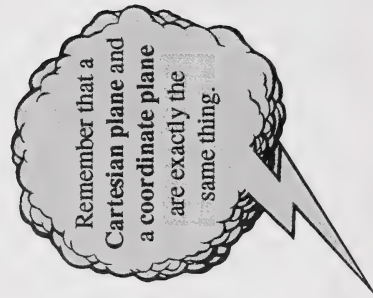
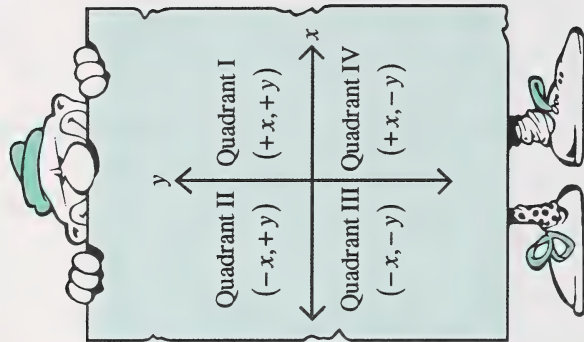


Given any point, you can always plot it in the coordinate plane.

Observe that the x - and y -axes are emphasized and that the points A , B , C , D , and E are clearly marked.

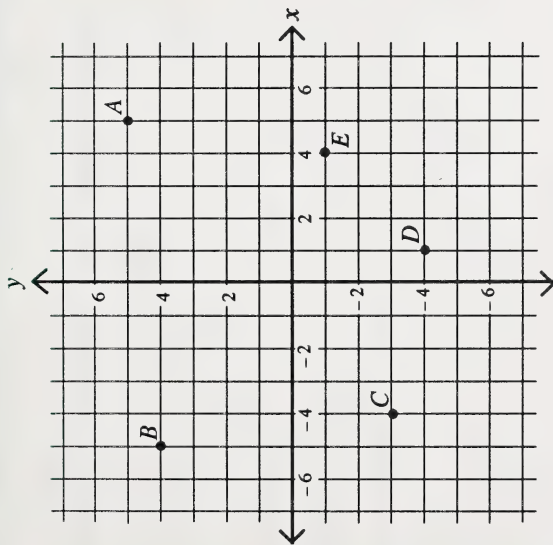
In this example, D lies in the first quadrant, A and E lie in the second quadrant, B lies in the third quadrant, and C lies in the fourth quadrant.

Points may lie on the axes. A point lying on the x -axis has a zero value for the y -part of the coordinate. For example, $F(4, 0)$ lies on the x -axis. A point lying on the y -axis has a zero value for the x -part of the coordinate. For example, $G(0, 5)$ lies on the y -axis.



Example 2

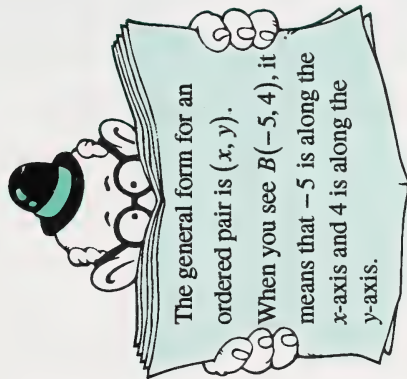
- In which quadrant is each point found?



Solution:

Point A is in Quadrant I.
Point B is in Quadrant II.
Point C is in Quadrant III.
Points D and E are in Quadrant IV.

Observe that $(1, -4)$ and $(4, -1)$ are two distinct points. Many students confuse points such as these.



- What are the coordinates of each point?

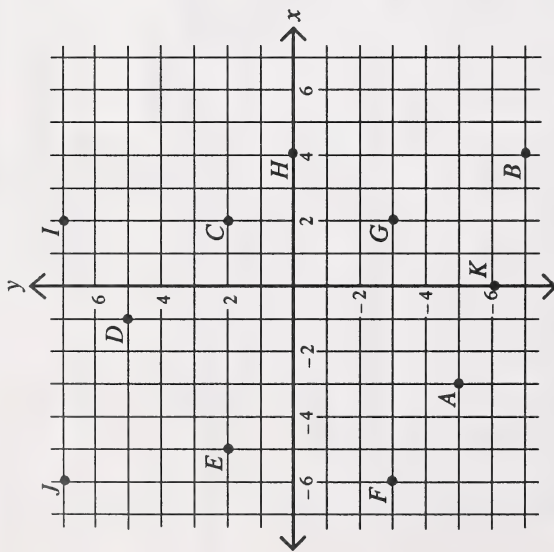
Solution:

In order to determine the x -coordinate for a point, count the number of units along the x -axis. Similarly, count the number of units along the y -axis to find the y -coordinate.

You will find that A is $(5, 5)$, B is $(-5, 4)$, C is $(-4, -3)$, D is $(1, -4)$, and E is $(4, -1)$.

Now try some questions.

Do any four of the following six questions. Questions 1, 2, and 3 are based on the following diagram.

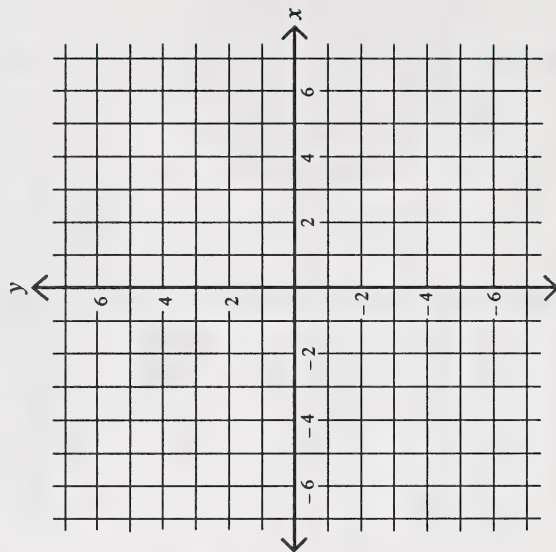


- What are the coordinates of the points A, B, C, D, and E?
- What point is named by each of the following ordered pairs?
 - $(-6, 7)$
 - $(-3, -5)$
 - $(2, 2)$
 - $(0, -6)$
 - $(2, 7)$
 - $(-5, 2)$
- Name the quadrant in which each point lies. What points are not in any of the quadrants?

- An ordered pair describes the coordinates of a point.
 - How can you tell when a point is on the x-axis?
 - How can you tell when a point is on the y-axis?
 - How can you tell when a point is not on any axis?
- Plot the following points.

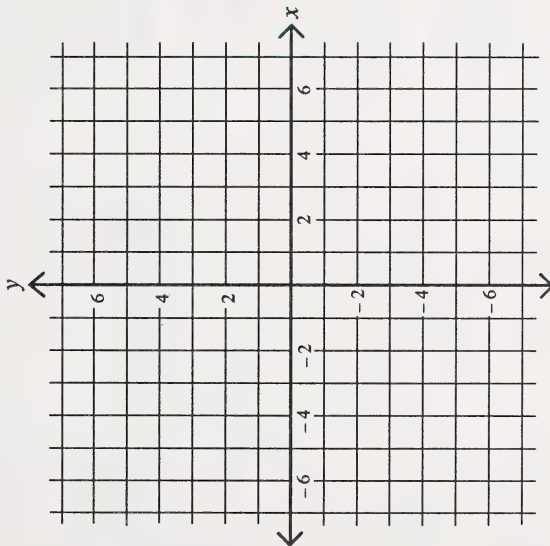
$L(-4, -3)$, $M(-2, 4)$, $N(0, -3)$, $O(2, 4)$, $P(4, -3)$

Join L to M, M to N, N to O, and O to P. What is the result when the points are joined in this way?



6. In which quadrant does each of these points lie? Plot the points on the following coordinate plane to check your answers.

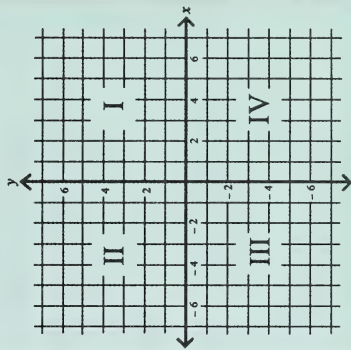
- | | | |
|----------------|----------------|----------------|
| a. $Q(-4, -5)$ | b. $R(3, 6)$ | c. $S(5, -3)$ |
| d. $T(1, 1)$ | e. $U(-6, 2)$ | f. $V(4, 1)$ |
| g. $W(-3, 7)$ | h. $X(-3, -4)$ | i. $Y(-5, -6)$ |
| j. $Z(2, -1)$ | | |



For solutions to Activity 1, turn to the Appendix, Topic 1.



Any Cartesian plane or coordinate plane is divided into four sections called quadrants.



If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

If you had difficulty understanding this topic, work through the following section.

Do **Part A** or **Part B**.



Part A

Use the computer diskette entitled *Problem Solving in Algebra*,¹ diskette 2, lessons 9 and 10 for additional help in naming and locating points on the Cartesian plane.



Part B

A **point** is considered to have no length, width, or thickness. It has no dimension. A point indicates position.

¹ *Problem Solving in Algebra* is a title of Britannica.

A **line** is one-dimensional. It has length but no width or thickness.

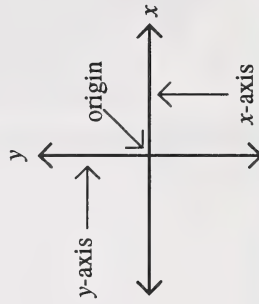
A **plane** is two-dimensional. It has length and width but no thickness. If this sheet did not have thickness, it would be a good example of a plane.

Space is three-dimensional. It has length, width, and thickness.

In this section you will consider the plane.

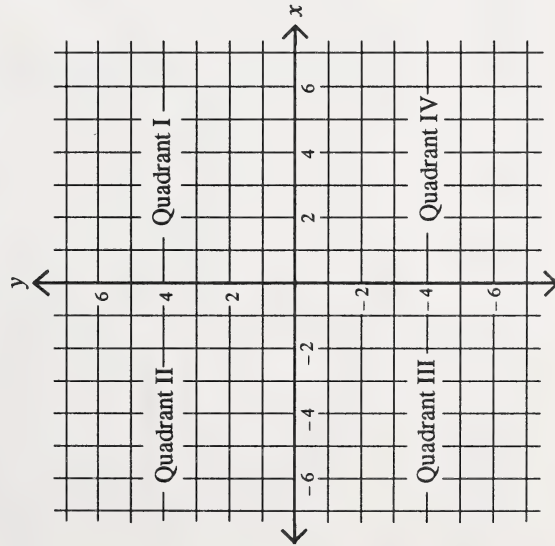
Take a sheet of paper. Draw two lines on the paper. One line should be vertical and the other should be horizontal. These two lines are perpendicular to each other.

Your sheet of paper **represents** the Cartesian plane. The horizontal line you drew is the **x-axis** and the vertical line is the **y-axis**. The point at which they cross or intersect is the **origin**. This diagram shows what you should have drawn.



You can use the Cartesian plane with more understanding if you follow some basic rules.

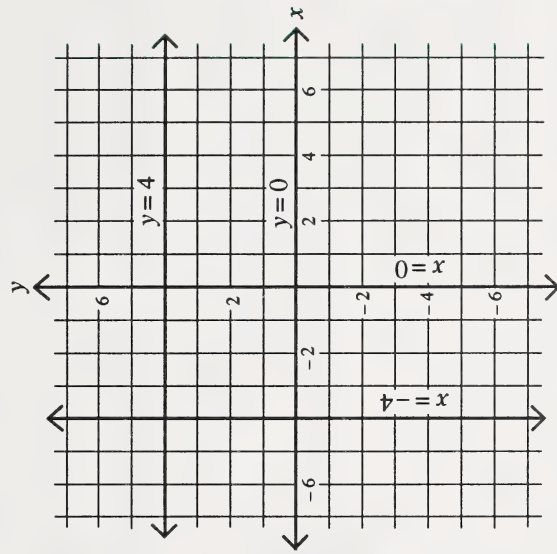
- Divide the plane into four **quadrants**. Start with the upper right section. Call this Quadrant I. Continue counterclockwise and locate Quadrant II, Quadrant III, and Quadrant IV.
- Place the x - and y -axes on a grid as shown. Consider the x -axis first. Start at the origin and number to the right with positive numbers and to the left with negative numbers. Now consider the y -axis. Number upward with positive numbers and downward with negative numbers.



- Name each axis in the Cartesian plane.

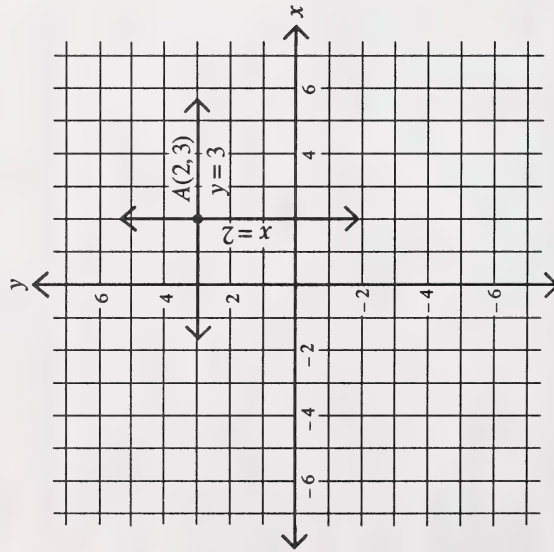
The x - and y -axes pass through the origin or $(0, 0)$. The x -axis is called $y = 0$. The y -axis is called $x = 0$. The vertical lines intersecting the x -axis will be $x = -2$, $x = -1$, $x = 1$, $x = 2$, and so on since they cross the axis at these points.

The same holds for the y -axis. Horizontal lines that pass through the y -axis will be $y = -2$, $y = -1$, $y = 2$, and so on since they cross the axis at these points.



In the Cartesian plane a point is represented by an **ordered pair** called the coordinate. In an ordered pair the first number is called the **abscissa** or **x-value**. The second number is called the **ordinate** or **y-value**.

The coordinate $(2, 3)$ at point A is written $A(2, 3)$. This implies that the lines $x = 2$ and $y = 3$ intersect at A.



Now try these questions.

1. Complete these statements by filling in the blanks.

- A _____ has no dimension. It has no length, width, or thickness.
- A line has _____ dimension. It has length but no _____ or _____.
- A _____ has length and width but no thickness. It has _____ dimensions.
- _____ has three dimensions. It has _____, _____, and _____.

2. What vertical and horizontal lines intersect at point $A(-4, 5)$?

3. Explain what is meant by each of the following:

- origin
- coordinates
- ordered pairs
- ordinate
- abscissa

4. What line is described by each of the following?

a. $x = 0$

b. $x = -4$

c. $y = 0$

d. $y = 3$

5. In what quadrant is each of the following points found?

a. $A(1, -4)$

b. $B(-4, -4)$

c. $C(-2, 4)$

d. $D(2, 4)$

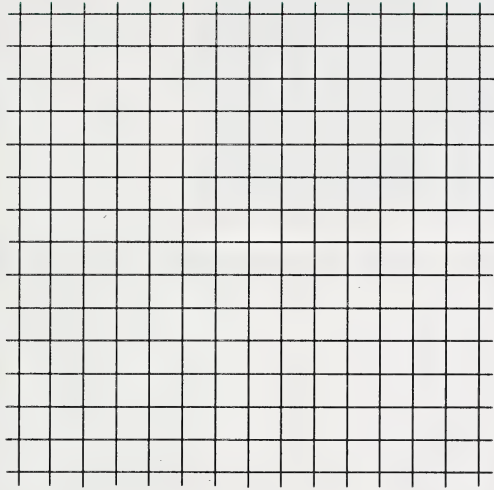
e. $E(5, -2)$

f. $F(-3, -5)$

g. $G(-4, 3)$

h. $H(5, 5)$

6. Using this grid, make a coordinate plane and plot the points from question 5.



For solutions to **Extra Help**, turn to the **Appendix, Topic 1**.



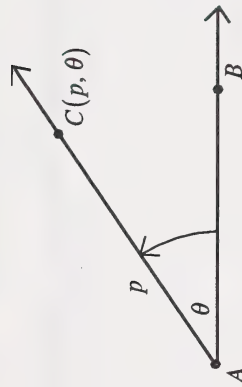
Extensions

In this unit you have learned about the Cartesian plane and the coordinates of points. You may be wondering if there are other ways of representing coordinates on a graph. The answer is yes. You can have a coordinate system whereby you locate points on a grid using an angle and a directed distance.

There are certain rules for this system:

- A reference line or axis must exist.
- All angles are measured in a counterclockwise direction or a clockwise direction.
- All distances are positive.

This system is called the polar coordinate system. The angle between the **initial arm** and the **terminal arm** is called θ . The length of the terminal arm is called p .



In this diagram the initial arm $\overrightarrow{AB}$ is the polar axis, the point A is the pole, and $\overrightarrow{AC}$ is the terminal arm.

Now study this example.

Example 3

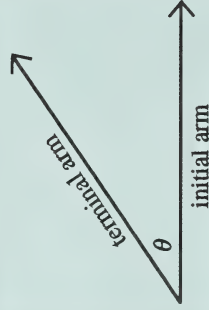
Plot the points with each of the following polar coordinates.

- $A(4, 0^\circ)$

Solution:



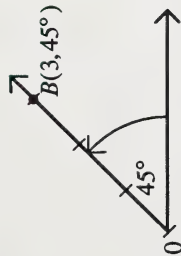
Note the following in this diagram:



- The initial arm is the base line of the angle.
- The terminal arm is the line which completes the angle as it rotates in a counterclockwise direction.
- The symbol θ is the Greek letter theta.

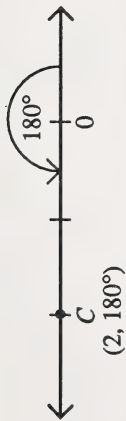
- $B(3, 45^\circ)$

Solution:



- $C(2, 180^\circ)$

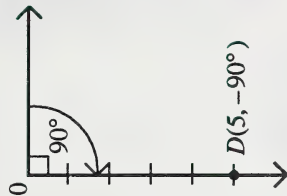
Solution:



Remember that in the polar coordinate system all distances are positive.

- $D(5, -90^\circ)$

Solution:



Notice that -90° would be the same as 270° .

You should note that all four points in Example 3 could be placed on one diagram. Observe also that for negative angles you measure in a clockwise direction.

Example 4

Give the polar coordinates for A, B, C, D, and E if they are points on the coordinate plane. Use the following information:

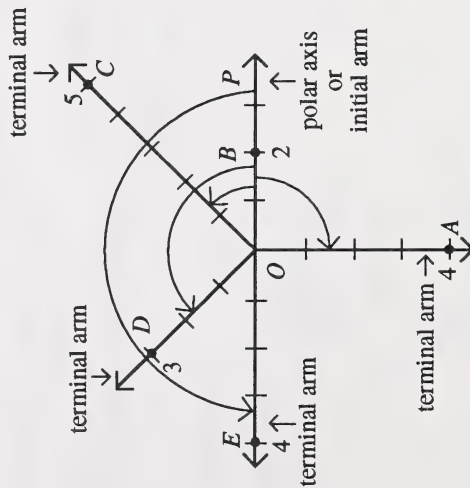
$$\angle BOC = 45^\circ$$

$$\angle BOD = 135^\circ$$

$$\angle BOE = 180^\circ$$

$$\angle BOA = -90^\circ$$

$$\angle BOP = 0^\circ$$



Solution:

$$A(4, -90^\circ)$$

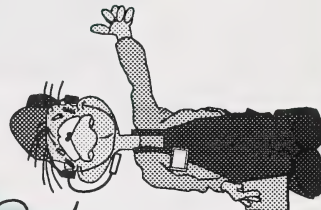
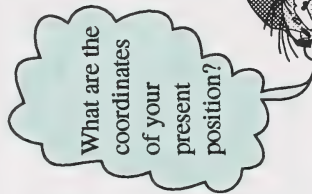
$$B(2, 0^\circ)$$

$$C(5, 45^\circ)$$

$$D(3, 135^\circ)$$

$$E(4, 180^\circ)$$

Polar coordinates are used to find bearings. Air traffic controllers and sea traffic controllers use polar coordinates to keep track of planes and ships, respectively.



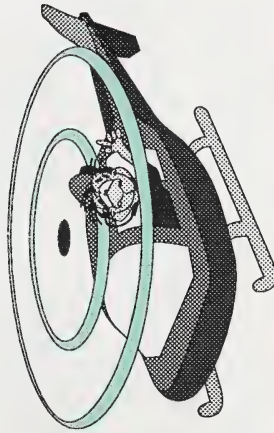
Do some of these questions.

1. Explain the following.

- pole
- polar axis
- terminal arm

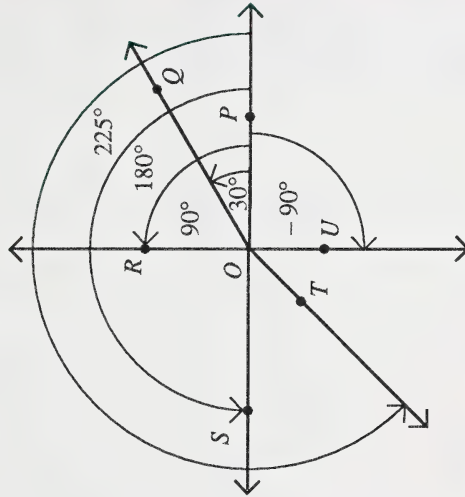
2. Plot the points with the following polar coordinates.

- $(3, 45^\circ)$
- $(4, 0^\circ)$
- $(2, 180^\circ)$
- $(5, 90^\circ)$
- $(3, -45^\circ)$



3. Given the following information, find the polar coordinates for the points on the diagram.

- $OP = 3.8$
 $OQ = 7$
 $OR = 4$
 $OS = 6$
 $OT = 2.5$
 $OU = 3$



For solutions to **Extensions**, turn to the **Appendix, Topic 1**.

Topic 2 Slope of a Line



Introduction

Many everyday situations involve the concept of **slope**. Some examples are the pitch of a roof, the incline of a stairway, and the position of a ladder leaning against a wall.

Engineers building a road or a railway through the mountains are concerned with the slope of the road bed and need to be able to calculate the vertical elevation or rise over a given horizontal distance or run.

In this topic you will learn to define a slope and use this new skill in solving practical problems.



What Lies Ahead

Throughout the topic you will learn to

1. use the slope formula to find positive, negative, zero, and undefined slopes
2. verify the relationships between the slopes of parallel and perpendicular lines and apply these to problem-solving situations

Now that you know what to expect, turn the page to begin your study of the slope of a line.



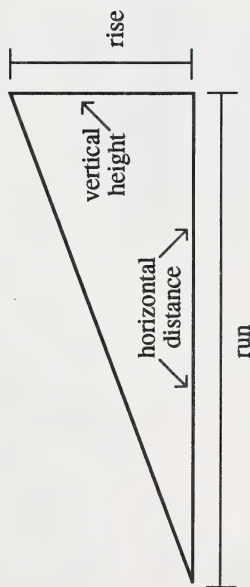
Exploring Topic 2

Activity 1



Use the slope formula to find positive, negative, zero, and undefined slopes.

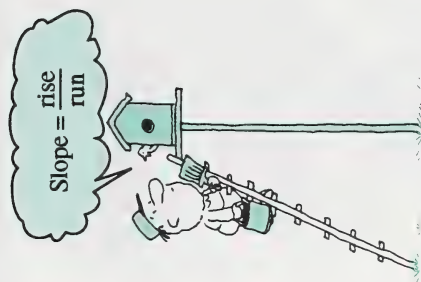
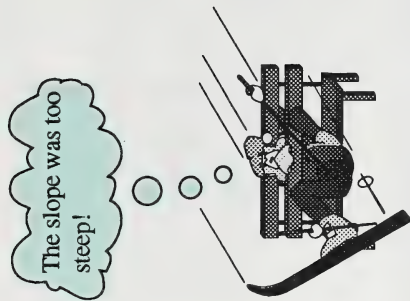
To determine the slope or steepness of an inclined plane, you compare the vertical height or rise to the horizontal distance or run as shown in the diagram.



The ratio of the amount of rise over the distance of the run gives you the measure of the slope.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

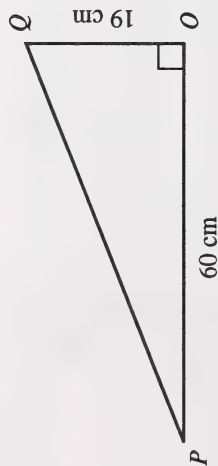
Study the next example.



Example 1

- Find the slopes of the following ramps.

Ramp 1



Ramp 2



Solution:

$$\begin{aligned}\text{Slope of Ramp 1} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{OQ}{OP} \\ &= \frac{19}{60}\end{aligned}$$

$$= 0.317 \text{ (rounded to three decimal places)}$$

To change $\frac{19}{60}$ to 0.317, you may use your calculator.



Enter	Display
19	19
$\div$	19
60	60
$=$	0.316666666

$$\frac{19}{60} \div 0.317 \text{ (rounded to three decimal places)}$$

$$\begin{aligned}\text{Slope of Ramp 2} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{YZ}{XZ} \\ &= \frac{17}{50} \\ &= 0.340 \text{ (rounded to three decimal places)}\end{aligned}$$

- Which ramp is steeper?

Solution:

Since 0.340 is greater than 0.317, Ramp 2 is steeper than Ramp 1.

When $\frac{19}{60}$ is changed to a decimal number, it is approximately 0.317.



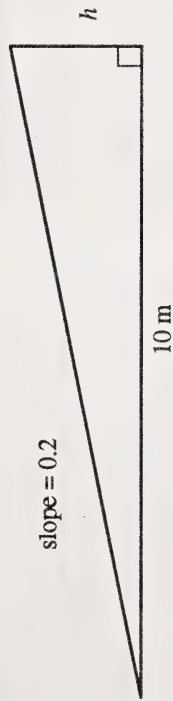
Did you notice that a slope is a ratio? It has no unit.

If you are given the slope and the rise, you can calculate the run. Similarly, if you are given the slope and the run, you can calculate the rise. The next example shows this.

Example 2

Duane walked up a ramp that has a slope of 0.2. If he moved 10 m in a horizontal direction, how many metres did he move up?

Solution:



Let h be the rise.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$0.2 = \frac{h}{10 \text{ m}}$$

$$h = 0.2(10 \text{ m})$$

$$= 2 \text{ m}$$

He moved up 2 m.

That slope wasn't very steep.



Always draw a diagram to represent the problem.



To solve for h in $\frac{0.2}{1} = \frac{h}{10 \text{ m}}$, cross multiply first.

$$0.2 = \frac{h}{10 \text{ m}}$$

$$1 \times h = 0.2(10 \text{ m})$$

$$h = 2 \text{ m}$$

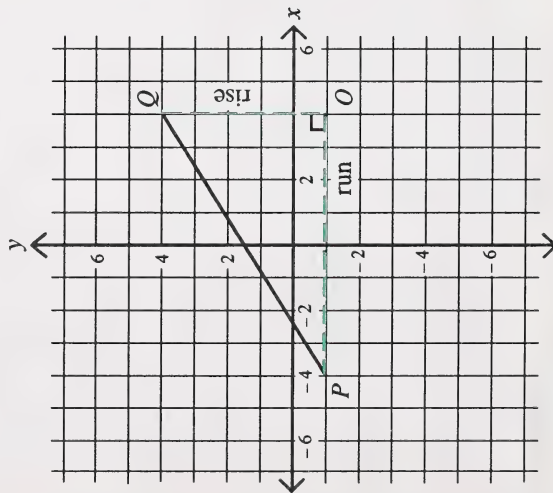


Do you remember the work you did on the coordinate plane? You will now use the coordinate system to calculate slope.

The coordinate system is a grid made up of many squares which has the x - and y -axes on it. You can calculate the rise and the run by counting squares. Once you have these values, you can calculate the slope. Look at the next example to see how this is done.

Example 3

Find the slope of the line segment PQ .



Solution:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

By counting squares, find the values for the rise and the run.

In order to get the rise, start at point O and count squares up to point Q . There are five squares. Similarly, for the run, start at point P and count squares to point O . There are eight squares.

If you let the letter m represent the slope, you can write the following:

$$\begin{aligned} \text{Slope } (m) &= \frac{\text{rise}}{\text{run}} \\ &= \frac{5}{8} \\ &= 0.625 \end{aligned}$$

The slope of line segment PQ is 0.625.

You can see that if you are given the coordinates of the points P and Q , you can draw the diagram, put in the rise and the run, and calculate the slope.



The rise and the run are easy to read from the coordinate plane. In this case the rise is 5 and the run is 8.

These two formulas are the same.

- $\text{Slope} = \frac{\text{rise}}{\text{run}}$
- $m = \frac{\text{rise}}{\text{run}}$

Notice that the symbol m is used for slope.



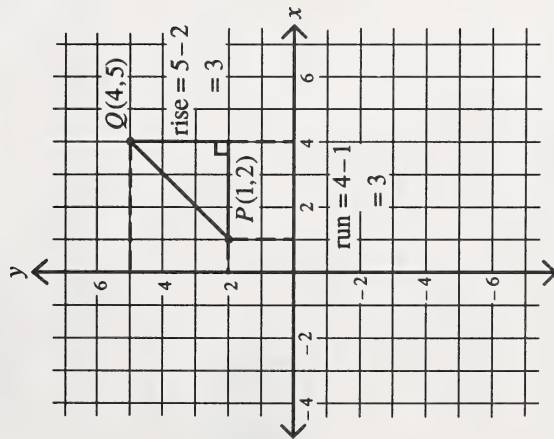
Another way to calculate the slope if you know the coordinates of the endpoints of a line segment is to calculate the rise and the run by **subtracting** the coordinates of the points.

Example 4

Find the slope of the line segment joining $P(1, 2)$ and $Q(4, 5)$.

Solution:

Draw a diagram.



$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ m &= \frac{5-2}{4-1} \\ &= \frac{3}{3} \\ &= 1\end{aligned}$$

The slope of line segment PQ is 1.

Do you see that the rise is the difference between the y-coordinates and the run is the difference between the x-coordinates?

It may be difficult in some cases to use just the graph to calculate the slope. You will now follow the development of a formula that can be used to find the slope directly from the coordinates.

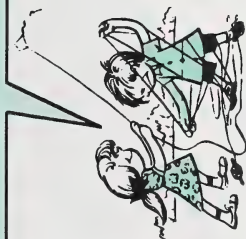
First choose any two points and use them to determine a formula for calculating slopes.

Use P_1 to denote one point on the coordinate plane. The coordinates of P_1 are (x_1, y_1) .

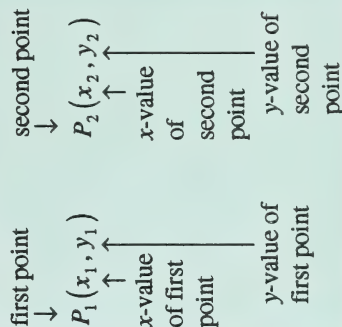
Use P_2 to denote the other point on the coordinate plane. The coordinates of P_2 are (x_2, y_2) .

You will now find the slope of the line segment P_1P_2 .

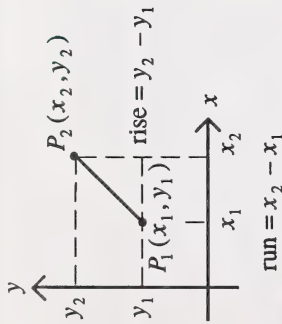
Endpoints are the two extreme points of any line segment. In $\overline{AB}$, A and B are the endpoints of line segment AB .



Instead of using different letters, subscripts are often used to distinguish between points and their coordinates.



Draw a diagram to represent the problem.



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\text{difference between } y\text{-coordinates}}{\text{difference between } x\text{-coordinates}}$$

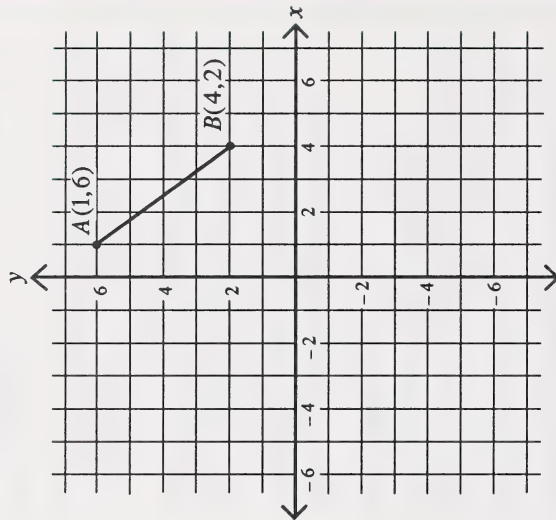
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Now apply this formula.

Example 5

Find the slope of the line segment joining

$A(1, 6)$ and $B(4, 2)$.



Solution:

Let $A(1, 6)$ be (x_1, y_1) .

$$\therefore x_1 = 1, y_1 = 6$$

Let $B(4, 2)$ be (x_2, y_2) .

$$\therefore x_2 = 4, y_2 = 2$$

From $A(1, 6)$, you get (x_1, y_1)
where $x_1 = 1$ and $y_1 = 6$.

From $B(4, 2)$, you get
 (x_2, y_2) where $x_2 = 4$ and
 $y_2 = 2$.

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{2 - 6}{4 - 1} \\
 &= \frac{-4}{+3} \\
 &= -\frac{4}{3}
 \end{aligned}$$

Note that the result would be the same if you let $B(4, 2) = (x_1, y_1)$ and $A(1, 6) = (x_2, y_2)$.

Then the slope would be calculated as follows:

$$\begin{aligned}
 m &= \frac{y_2 - y_1}{x_2 - x_1} \\
 &= \frac{6 - 2}{1 - 4} \\
 &= \frac{+4}{-3} \\
 &= -\frac{4}{3}
 \end{aligned}$$

Changing the order of the points does not affect the answer obtained for the slope.

Why is the slope negative in the previous example? Read on and find out.

If you look at the first four examples, the line segments all **rise** to the right. The slope is **positive** in each case. In Example 5 the line segment **falls** to the right and its slope is **negative**. The sign of the slope, therefore, indicates whether a line rises or falls.

line rises to the right



positive slope (+)

line falls to the right



negative slope (-)



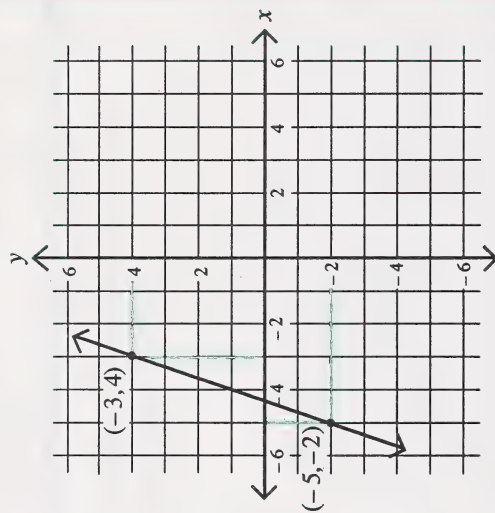
Example 6

Find the slope of the line through $(-3, 4)$ and $(-5, -2)$ and state whether the line rises or falls to the right.

Solution:

Let $(-3, 4) = (x_1, y_1)$ and $(-5, -2) = (x_2, y_2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{-2 - 4}{-5 - (-3)} \\ &= \frac{-6}{-5 + 3} \\ &= \frac{-6}{-2} \\ &= 3 \end{aligned}$$



The slope is positive. You can see in the diagram of the coordinate plane that the line rises to the right.

In Example 6 you should be able to see instantly that the slope is positive since the line rises to the right.



Watch your signs! Since you are subtracting a negative number in the denominator $-5 - (-3)$, the parentheses are used. When the parentheses are removed, the sign of the number being subtracted is changed to its opposite and the subtraction sign is changed to an addition sign.

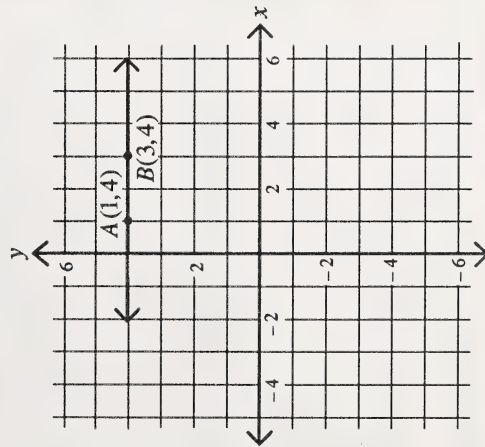
Lines that have a negative slope or a positive slope are called **oblique**. Not all lines have a negative slope or a positive slope. Vertical and horizontal lines fall into this category. You will learn more about these now. Look at the next two examples.

Example 7

Calculate the slope of the line through $A(1, 4)$ and $B(3, 4)$.

Solution:

A sketch of this problem reveals that the line is horizontal.



Let $A(1, 4) = (x_1, y_1)$ and $B(3, 4) = (x_2, y_2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 4}{3 - 1} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$

The slope of the line is zero.

For a horizontal line, the slope is as follows:

$$\begin{aligned} \text{Slope} &= \frac{0}{\text{run}} \\ &= 0 \end{aligned}$$



Remember that an oblique line is a line which is slanted to the right or to the left.

The next example involves the slope of a vertical line.

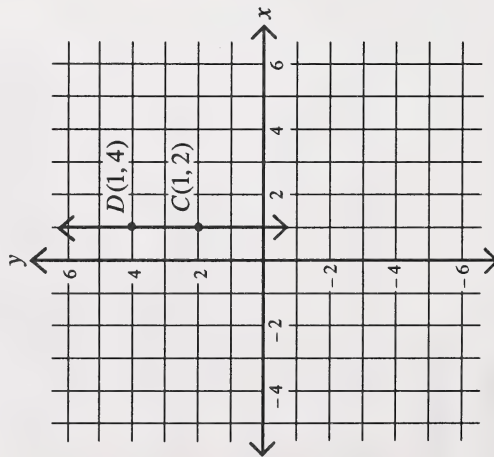
Example 8

Find the slope of the line through $C(1, 2)$ and $D(1, 4)$.

Solution:

A sketch shows that the line is vertical.

The line is vertical since each point is one unit from the y -axis.



Let $C(1, 2) = (x_1, y_1)$ and $D(1, 4) = (x_2, y_2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{4 - 2}{1 - 1} \\ &= \frac{2}{0} \end{aligned}$$

= undefined

The slope of the line is undefined.

For a vertical line, the slope is as follows:

$$\begin{aligned} \text{Slope} &= \frac{\text{rise}}{0} \\ &= \text{undefined} \end{aligned}$$



If the denominator of a fraction is zero, the value of the fraction is undefined since division by zero is undefined. Thus, $\frac{a}{b}$ is undefined if $b = 0$.

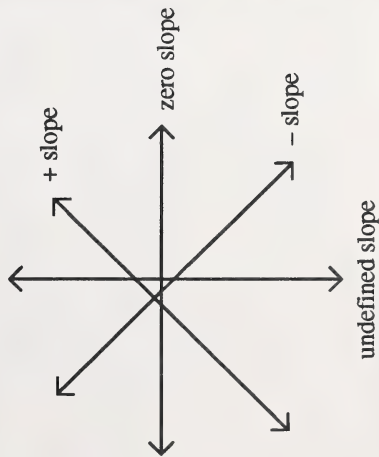
In summary, the important ideas concerning the slope of a line are as follows:



$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

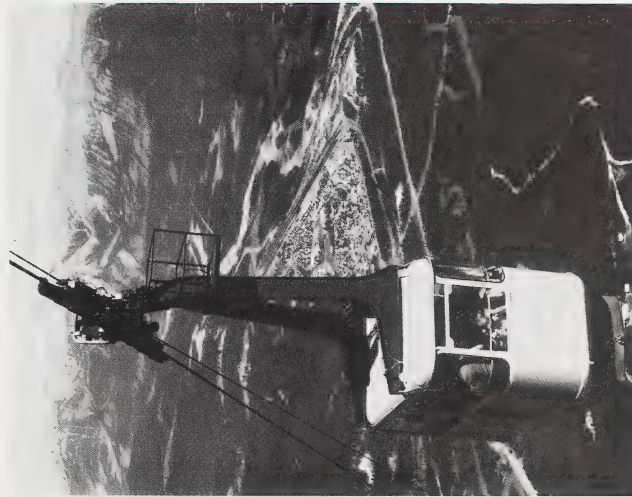
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope	Line
positive	rises to the right
zero	horizontal
negative	falls to the right
undefined	vertical

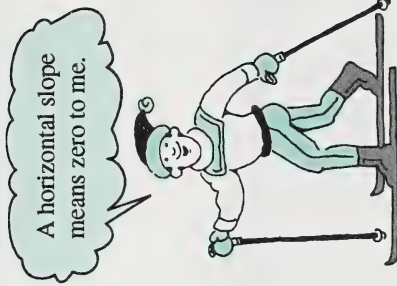


You may go to the computer program entitled *Algebra Drill and Practice I*, Part 3, Section 1 on finding the slope of a line given two points. Follow some of the examples and do as many questions as you feel are necessary.

Now try some of the following questions.

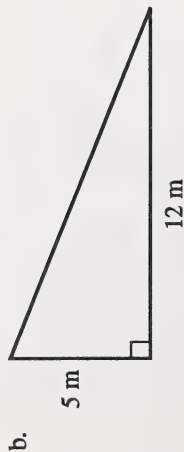
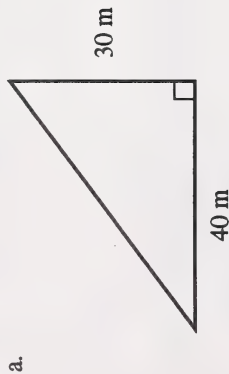


¹ *Algebra Drill and Practice I* is a title of Conduit.

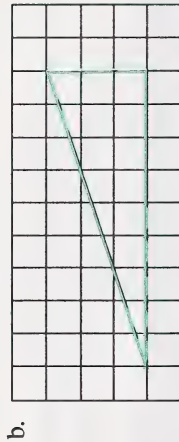
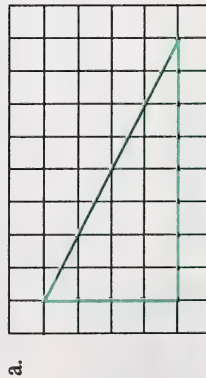


Do any five of the following nine questions.

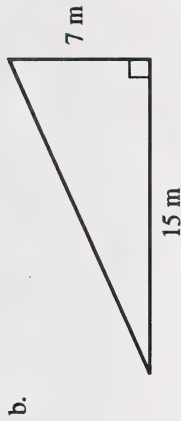
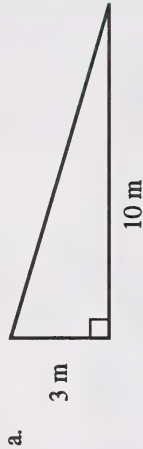
- From the information given, find the slope of each incline. Round your answer in b to three decimal places.



- Using the method of counting squares, find the slopes of the ramps in each of the following grids.



- The slope of the roof of a building is called the pitch. The following diagrams represent sections of the pitch of a shed. Find the pitch in each case. Round your answer in b to three decimal places.

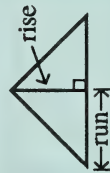


Remember that a slope is a ratio such as $\frac{3}{4}$ or 0.75. It has no unit.



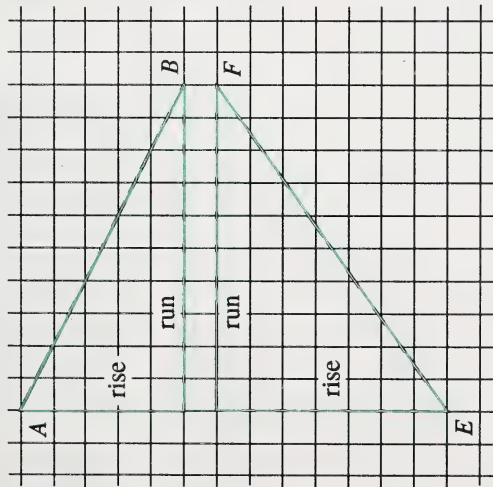
The pitch of a roof is given by the following:

$$\text{Pitch} = \frac{\text{rise}}{\text{run}}$$

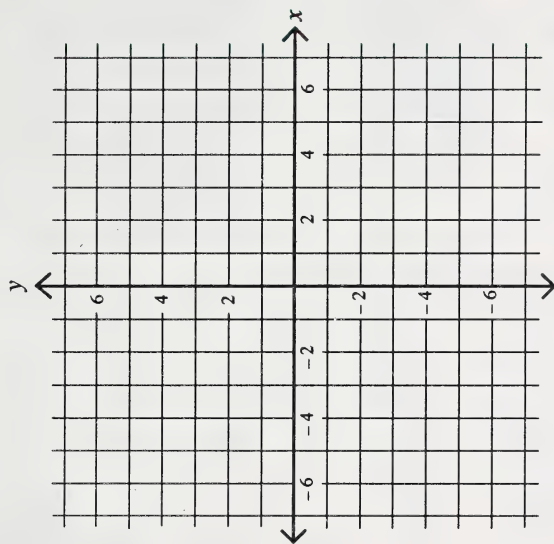


Pitch is always positive.

4. Find the slope of $\overline{AB}$. Find the slope of $\overline{EF}$. Then compare the two slopes.

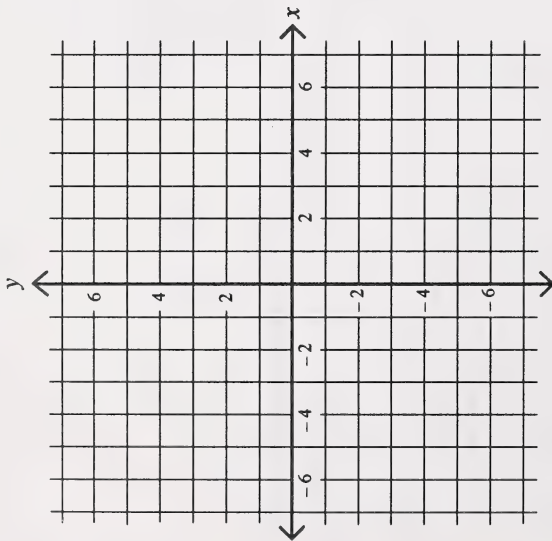


6. a. Plot $D(5, 1)$ and $E(-4, 3)$ on the coordinate plane. Join the points to form a line segment. Calculate the slope using the formula.

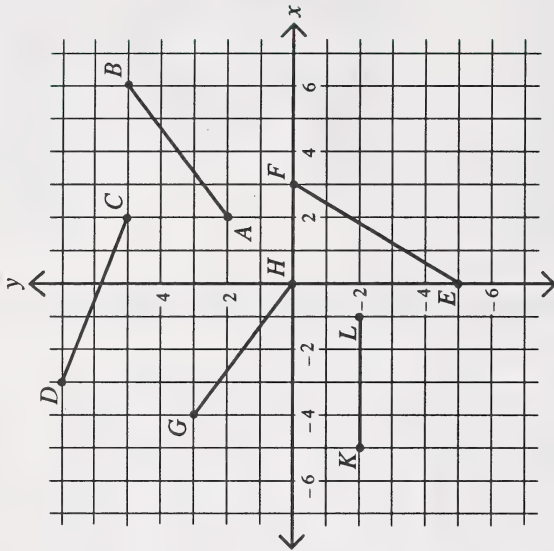


5. The two endpoints of the line segment PQ are $P(2, 5)$ and $Q(-2, 2)$. What is the slope of the line segment?

- b. Plot $K(-5, 2)$ and $L(1, -3)$ on the coordinate plane. Join the points to form a line segment. Calculate the slope using the formula.



7. By counting the squares, calculate the slope of each line segment in the following coordinate plane.



8. The slope of a line is zero. The run is 8 cm.
- What is the rise?
 - What type of line is this?
9. The run of a ramp is 2.5 cm. The slope is one. What is the rise?



For solutions to **Activity 1**, turn to the **Appendix**,
Topic 2.

Activity 2

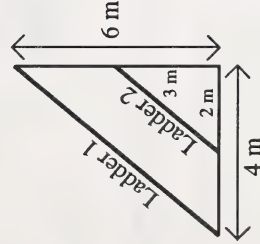


Verify the relationships between the slopes of parallel and perpendicular lines and apply these to problem-solving situations.

If you attend the annual air shows in Red Deer or Edmonton, Alberta, you will observe planes flying in various formations. When they are flying side by side and leave vapour trails behind, you will see vapour trails the same distance apart. These trails are parallel.

A big ladder and a small ladder are leaning against a vertical wall. These ladders appear to be parallel to each other. Are they parallel?

Consider a sketch of the ladders leaning against the wall. The distances from the foot of the wall to the endpoints of the ladders are given.



You may use the fact that slope = $\frac{\text{rise}}{\text{run}}$ to compare the two slopes.

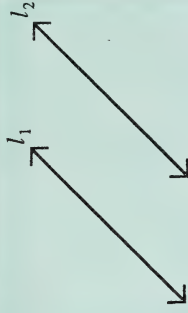
$$\begin{aligned}\text{Slope of Ladder 1} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{6 \text{ m}}{4 \text{ m}} \\ &= 1.5\end{aligned}$$

$$\begin{aligned}\text{Slope of Ladder 2} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3 \text{ m}}{2 \text{ m}} \\ &= 1.5\end{aligned}$$

Since the ladders have the same slope, they must be parallel.



The slopes of parallel lines are equal. If two lines have the same slope, they must be parallel.



Parallel lines will never intersect. The symbol $\parallel$ is used to indicate that lines are parallel. In this case $l_1 \parallel l_2$.

Parallel lines have the same slope.



Subscripts are used to distinguish different slopes and different lines.

The line l_1 would have a slope of m_1 .

The line l_2 would have a slope of m_2 .

The line l_3 would have a slope of m_3 .

You should see a pattern that you can apply to different situations.

Look at these lines.



The slope of l_1 is m_1 . The slope of l_2 is m_2 .

If $l_1 \parallel l_2$, then $m_1 = m_2$.

If $m_1 = m_2$, then $l_1 \parallel l_2$.

Study this example.

Example 9

Consider the points $A(-3, 1)$, $B(5, 8)$, $P(-2, -2)$, and $Q(6, 5)$ which are in the same plane. Show that line segments AB and PQ are parallel.

Solution:

Let m_1 be the slope of line segment AB and m_2 be the slope of line segment PQ .

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m_1 &= \frac{1 - 8}{-3 - 5} \\ &= \frac{-7}{-8} \\ &= \frac{7}{8} \end{aligned}$$

$$\begin{array}{cc} A(-3, 1) & \text{and } B(5, 8) \\ \downarrow & \downarrow \\ (x_2, y_2) & (x_1, y_1) \end{array}$$

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ m_2 &= \frac{-2 - 5}{-2 - 6} \\ &= \frac{-7}{-8} \\ &= \frac{7}{8} \end{aligned}$$

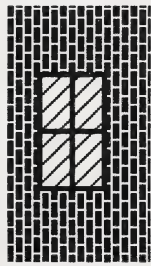
$$\begin{array}{cc} P(-2, -2) & \text{and } Q(6, 5) \\ \downarrow & \downarrow \\ (x_2, y_2) & (x_1, y_1) \end{array}$$

Since $m_1 = m_2$, line segment AB must be parallel to line segment PQ .

Now you will look at **perpendicular lines**. You should remember that lines which intersect at right angles are perpendicular lines.

Some examples of perpendicular lines follow.

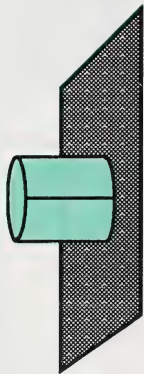
The corners of a window form angles which measure 90° each. Adjacent sides of the window are perpendicular to each other. This makes the corners form right angles.



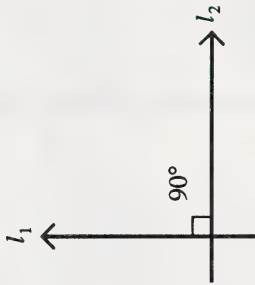
A flagpole is perpendicular to its shadow.



A line drawn down the middle of a can is perpendicular to the flat surface of a table.

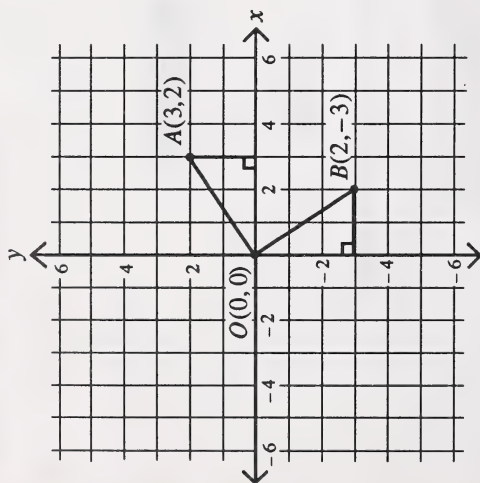


Remember that perpendicular lines cross at right angles.

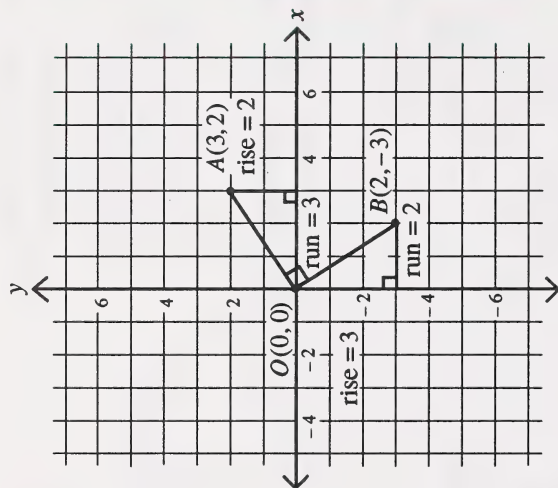


The symbol $\perp$ is used to indicate that lines are perpendicular. In this case $l_1 \perp l_2$.

Now you will compare the slopes of lines that are perpendicular to each other. Point A and point B have been selected such that $\overline{OA} \perp \overline{OB}$ as shown in the diagram.



Now you can calculate the slopes of $\overline{OA}$ and $\overline{OB}$.



The slope of $\overline{OA}$ is $\frac{2}{3}$.

The slope of $\overline{OB}$ is $-\frac{3}{2}$.

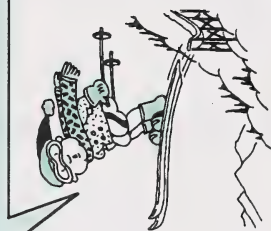
Compare the slopes of the two line segments. Note that $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals. When multiplied, the product of the two slopes is -1 .

$$\frac{2}{3} \times -\frac{3}{2} = -1$$

You can calculate the slopes of $\overline{OA}$ and $\overline{OB}$ using the slope formula.

$$\begin{aligned} \text{Slope of } \overline{OA} &= \frac{2-0}{3-0} \\ &= \frac{2}{3} \end{aligned}$$

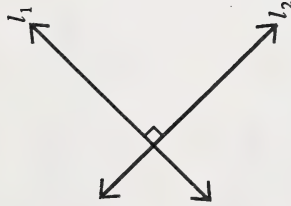
$$\begin{aligned} \text{Slope of } \overline{OB} &= \frac{-3-0}{2-0} \\ &= -\frac{3}{2} \text{ or } -\frac{3}{2} \end{aligned}$$



Remember that $\frac{a}{b}$ and $\frac{b}{a}$ are called reciprocals since their product is 1. The values $\frac{2}{3}$ and $-\frac{3}{2}$ are negative reciprocals since their product is -1 .



You have now discovered that the slopes of perpendicular lines are negative reciprocals. In reverse, if the slopes of two lines are negative reciprocals, then the lines are perpendicular.



If $l_1 \perp l_2$, the slope of $l_1 = m_1$, and the slope of $l_2 = m_2$, then the following is true:

$$\begin{aligned} m_1 \times m_2 &= -1 \\ \text{or } m_1 &= \frac{-1}{m_2} \\ \text{or } m_2 &= \frac{-1}{m_1} \end{aligned}$$

This information is only true for oblique lines.

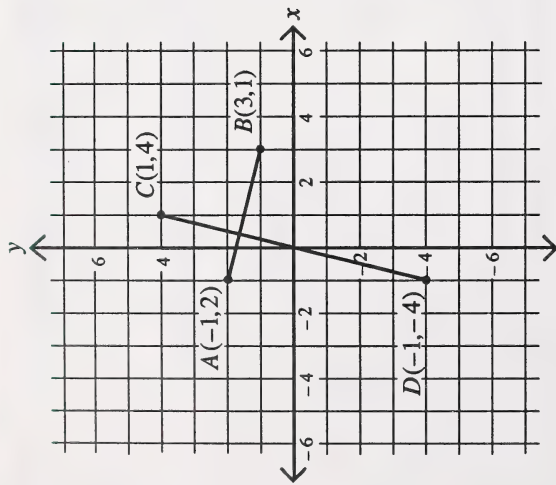
Now study these examples involving perpendicular lines.

Example 10

Show that the line segments AB and CD are perpendicular if $A(-1, 2)$ and $B(3, 1)$ are the endpoints of one line segment and $C(1, 4)$ and $D(-1, -4)$ are the endpoints of the other line segment.

Solution:

It is important to sketch a diagram. Plot the points A, B, C , and D . Join A to B and C to D . Do the segments look perpendicular?



The slope of $\overline{CD}$ is as follows:

$$\begin{aligned} m &= \frac{4 - (-4)}{1 - (-1)} \\ &= \frac{4 + 4}{1 + 1} \\ &= \frac{8}{2} \\ &= \frac{4}{1} \end{aligned}$$

Because the product of the slopes $\left(\frac{-1}{4} \times \frac{4}{1}\right) = -1$, $\overline{AB}$ and $\overline{CD}$ are perpendicular.

The slope of $\overline{AB}$ is as follows:

$$\begin{aligned} m &= \frac{1 - 2}{3 - (-1)} \\ &= \frac{-1}{3 + 1} \\ &= \frac{-1}{4} \text{ or } -\frac{1}{4} \end{aligned}$$

Point A $(-1, 2)$
↓
 (x_1, y_1)

Point B $(3, 1)$
↓
 (x_2, y_2)

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1 - 2}{3 - (-1)}$$

$$= \frac{-1}{4} \text{ or } -\frac{1}{4}$$

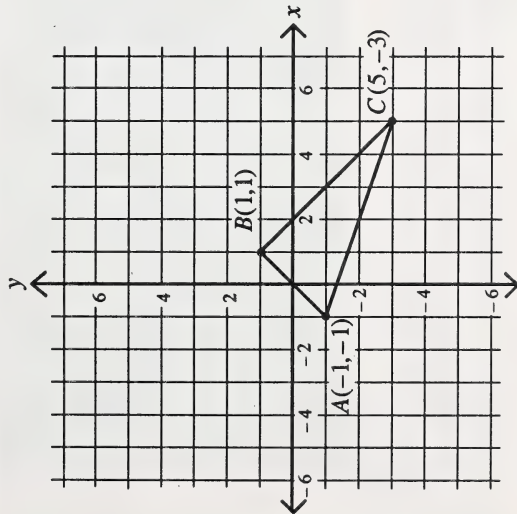


You will discover in later courses that there is extensive use made of the fact that $m_1 \times m_2 = -1$. It is used to establish whether a triangle is a right triangle and it also determines right-angle properties. You should try an example of this type.

Example 11

Establish the fact that triangle ABC is a right triangle if the vertices are $A(-1, -1)$, $B(1, 1)$, and $C(5, -3)$.

Solution:



Which two sides look perpendicular? Since $\overline{AB}$ looks perpendicular to $\overline{BC}$, find the slopes of these two sides first.

$$\begin{aligned}\text{Slope of } \overline{AB} &= \frac{1 - (-1)}{1 - (-1)} \\ &= \frac{1+1}{1+1} \\ &= \frac{2}{2} \\ &= 1\end{aligned}$$

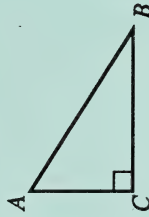
$$\begin{aligned}\text{Slope of } \overline{BC} &= \frac{-3 - 1}{5 - 1} \\ &= \frac{-4}{4} \\ &= -1\end{aligned}$$

$$\begin{aligned}\text{Slope of } \overline{AB} \times \text{Slope of } \overline{BC} &= 1(-1) \\ &= -1\end{aligned}$$

$$\therefore \overline{AB} \perp \overline{BC}$$

It follows that $\triangle ABC$ is a right triangle.

This is a right triangle since it has two sides which are perpendicular.



In triangle ABC , $\overline{AC} \perp \overline{BC}$.

- If line segments are perpendicular, the product of their slopes is -1 .
- In Example 11 try the sides that look perpendicular first. Then, find the slope of the other side. Check all slopes until you find the right pair. If no slopes are of the relation $m_1 m_2 = -1$, then the triangle is not a right triangle.
- The slopes of $\overline{AB}$ and $\overline{BC}$ are found using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Review this section, and then try some of the following questions.

Do any five of the following questions.

1. Complete the table. Assume m_1 and m_2 are the slopes of perpendicular lines. (Hint: To find the slope of the second line, simply write the negative reciprocal of the slope of the first line.)

m_1	m_2	$m_1 \times m_2$
2		
	-3	
$\frac{2}{5}$		
	0	

2. Given $A(-1, 3)$, $B(4, -4)$, $C(5, 3)$, and $D(-2, -2)$, are $\overline{AB}$ and $\overline{CD}$ perpendicular?
3. Show that $\triangle ABC$ is a right triangle given $A(0, -1)$, $B(3, 2)$, and $C(4, 1)$.
4. Two lines are perpendicular if $m_1 m_2 = -1$. When is this statement not true? Why?
5. Calculate the slope of the line segment joining each pair of points.
 - a. $P(4, 5)$, $Q(2, 10)$
 - b. $M(-7, 3)$, $N(2, -1)$
 - c. $A(0, 6)$, $B(0, 4)$
 - d. $C(-2, -4)$, $D(-8, -1)$
 - e. $F(9, 0)$, $G(-5, 0)$

6. If $m_1 m_2 = -1$, find the value of m_2 for each of the following.

a. $m_1 = \frac{3}{4}$

b. $m_1 = 0$

c. $m_1 = -\frac{3}{5}$

d. $m_1 = 3$

e. $m_1 = 1$

7. If the following are the endpoints of line segments, which of the line segments are parallel?

a. $C(7, 2), D(5, 9)$

b. $X(2, -1), Y(-7, 3)$

c. $A(4, 9), B(1, 3)$

d. $M(4, -6), N(-3, -2)$

e. $R(2, 8), S(4, 1)$

f. $P(4, 5), Q(2, 1)$

8. A line passes through the points $P(6, 6)$ and $Q(2, 5)$. Show that it is parallel to the line which passes through $A(3, -5)$ and $B(7, -4)$.



For solutions to Activity 2, turn to the **Appendix, Topic 2**.



Remember these important points:

- To calculate the slope of a line segment, you can use the following formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

- Two lines are parallel if their slopes are equal.

- Two lines are perpendicular if the product of their slopes is -1 .



If you require help, do the Extra Help section.

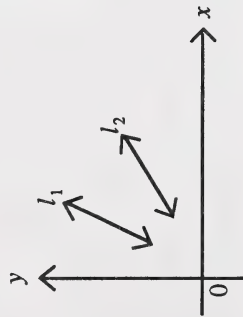
If you want more challenging explorations, do the Extensions section.

You may decide to do both.



Extra Help

You may speak of the steepness of one line as opposed to another. Look at the diagram.



You could say l_1 is steeper than l_2 .

A measure of the steepness of a line is called slope. By definition, slope is as follows:

$$\text{Slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{vertical change}}{\text{horizontal change}}$$

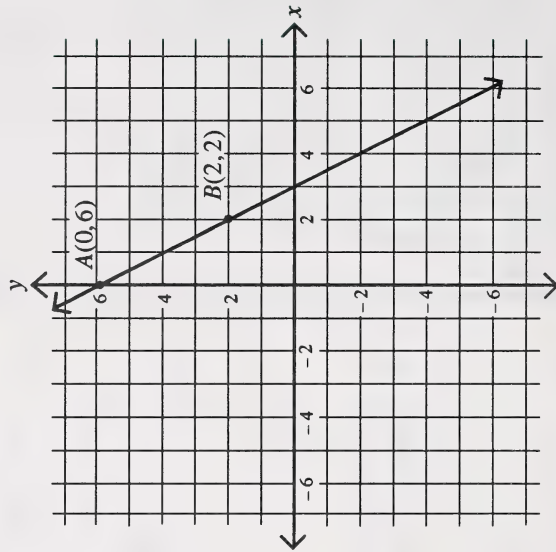
If a line runs parallel to the x -axis, the rise would be zero. The slope would be zero also.

If a line runs parallel to the y -axis, the run would be zero. The slope would be undefined because you cannot divide by zero.

Example 12

Find the slope of a line through the points $A(0, 6)$ and $B(2, 2)$.

Solution:



$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2-6}{2-0} \\ &= \frac{-4}{2} \\ &= -2\end{aligned}$$

The slope of $\overleftrightarrow{AB}$ is -2 .

Example 13

If P is $(2, 8)$, Q is $(2, 2)$, and R is $(6, 2)$, show that $\overline{PQ}$ is vertical and $\overline{QR}$ is horizontal.

Solution:

$$\begin{aligned}\text{Slope of } \overline{PQ} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2-8}{2-2} \\ &= \frac{-6}{0} \\ &= \text{undefined}\end{aligned}$$

Therefore, $\overline{PQ}$ is vertical.

$$\begin{aligned}\text{Slope of } \overline{QR} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{2-2}{6-2} \\ &= \frac{0}{4} \\ &= 0\end{aligned}$$

Therefore, $\overline{QR}$ is horizontal.

The slope of l_1 is m_1 and the slope of l_2 is m_2 .

If l_1 is parallel to l_2 , then $m_1 = m_2$.

If l_1 is perpendicular to l_2 , then $m_1 \times m_2 = -1$.

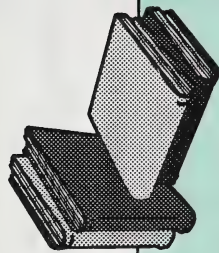
Example 14

- The slope of l_1 is $-\frac{3}{2}$. If l_1 is parallel to l_2 , what would the slope of l_2 be?

Solution:

If $l_1 \parallel l_2$, the slope of l_1 = the slope of l_2 .

Therefore, the slope of l_2 is $-\frac{3}{2}$.



- Vertical lines have a slope which is undefined.
- Horizontal lines have a slope of zero.

- Line l_3 is perpendicular to line l_1 . If the slope of l_1 is $-\frac{3}{2}$, what would the slope of l_3 be?

Solution:

$$\text{Slope of } l_1 = m_1 = \frac{-3}{2}$$

$$\text{If } l_1 \perp l_3, \text{ then } m_1 \times m_3 = -1.$$

$$\left(\frac{-3}{2}\right)(m_3) = -1$$

$$\begin{aligned} m_3 &= -1 \div \frac{-3}{2} \\ &= -1 \times \frac{2}{-3} \\ &= \frac{2}{3} \end{aligned}$$

The slope of l_3 is $\frac{2}{3}$.

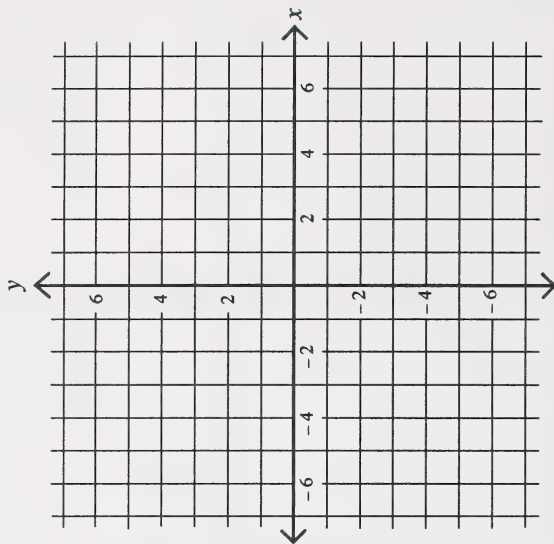
Check:

The slopes $-\frac{3}{2}$ and $\frac{2}{3}$ are indeed negative reciprocals because

$$-\frac{3}{2} \times \frac{2}{3} = \frac{-6}{6} = -1.$$

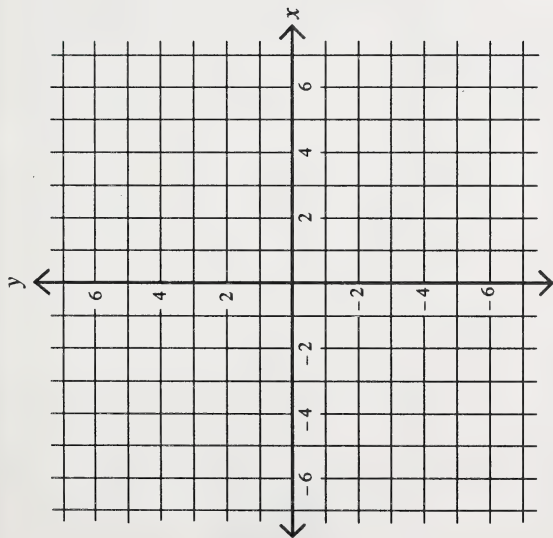
Now try the following questions.

- Given $A(-2, 3)$ and $B(4, 3)$, draw the graph of line AB .



- What do you notice about line AB when it is drawn on a coordinate plane?
- What is the slope of $\overleftrightarrow{AB}$?
- Make a general statement about the slope of any line parallel to the x -axis.

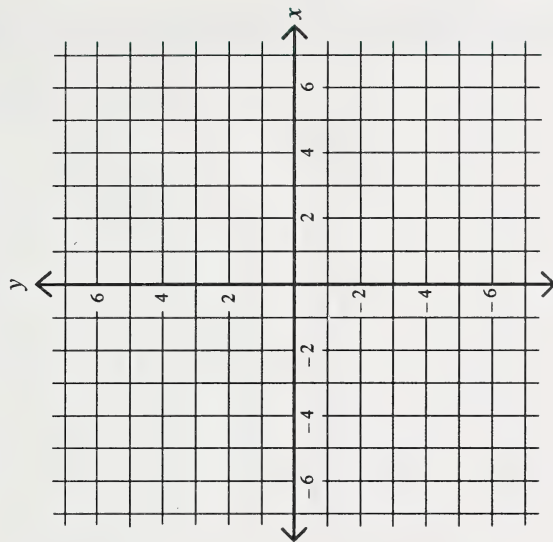
2. a. Given $P(4, -2)$ and $Q(4, 4)$, draw a graph of line PQ .



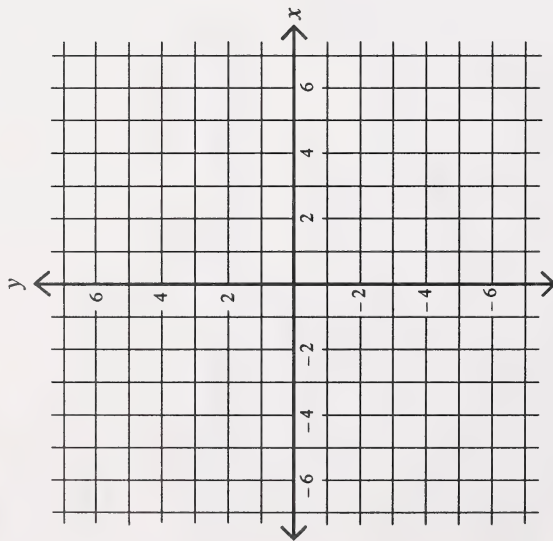
- b. What is the slope of $\overleftrightarrow{PQ}$?

- c. Why do you say that the slope of a line parallel to the y-axis is undefined?

3. Determine whether the line through $A(1, 4)$ and $B(5, 5)$ is parallel to the line through $C(3, -3)$ and $D(7, -2)$. Draw the graph.



4. Show that the line through $R(1, 6)$ and $S(3, 5)$ is perpendicular to the line through $T(-4, 1)$ and $U(-1, 7)$. Draw the graph.



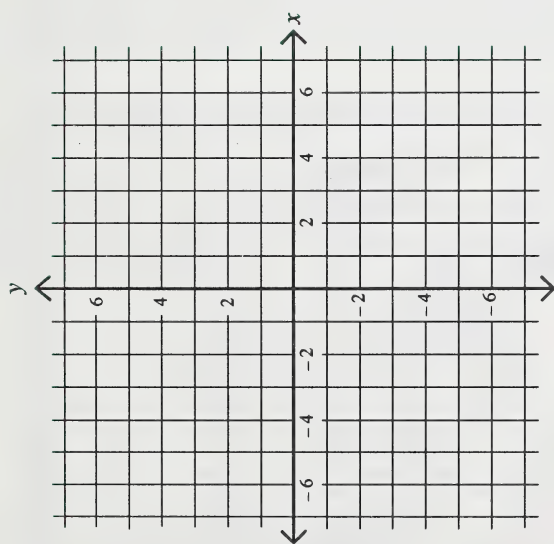
For solutions to **Extra Help**, turn to the **Appendix**, **Topic 2**.



Extensions

You may want to try the following questions to challenge your knowledge of slope.

- Two points $A(-2, 3b)$ and $B(b, 4)$ determine a line. If the slope of the line is $\frac{1}{3}$, what are the coordinates of A and B ?
- Two vertices of $\triangle ABC$ are $C(1, 3)$ and $B(5, 18)$. If $D(d, 6)$ is a point on $\overline{BC}$, what is the value of d ?
- The vertices of a figure are $P(-2, 0)$, $Q(6, 5)$, $R(1, -2)$ and $S(-7, -7)$.
 - Plot the points on the following coordinate plane and connect P to Q , Q to R , R to S , and S to P .
 - Calculate the slope of each side.
 - What do you notice about the slopes of the sides?
 - What type of polygon is figure $PQRS$?



4. a. Calculate the slopes of $\overline{PQ}$ and $\overline{RS}$ given the points $P(3, -4)$, $Q(9, 2)$, $R(-5, 1)$, and $S(5, 3)$.
- b. Are the line segments PQ and RS parallel or perpendicular?



For solutions to Extensions, turn to the **Appendix, Topic 2**.

Topic 3 Graphing Linear Equations



Introduction

When a quarterback throws a football, it is intended for an eligible receiver on the team.



Many times a defensive player of the opposing team cuts across the path of the football and interferes with the ball in flight. If the opposing player actually catches the ball, the ball is said to be intercepted.

Intercepts occur in graphing too. In this topic you will see how these occur in relation to the x - and y -axes.



What Lies Ahead

Throughout the topic you will learn to

1. plot several points and draw the graph of a linear equation
2. write and graph the equation of a line in the form $y = mx + b$

Now that you know what to expect, turn the page to begin your study of graphing linear equations.



Exploring Topic 3



In a linear equation all numerical coefficients cannot equal zero.

Activity 1



Plot several points and draw the graph of a linear equation.

A linear equation is a statement of the form $Ax + By + C = 0$ where A , B , and C are numerical coefficients and at least one of A or B is not equal to zero.

For example, $3x + 2y + 7 = 0$ is a linear equation. Here $A = 3$, $B = 2$, and $C = 7$.

If $A = 0$, $B = 2$, and $C = 7$, $Ax + By + C = 0$ becomes $0x + 2y + 7 = 0$ or $2y + 7 = 0$. This is an acceptable equation for a line.

If $A = 3$, $B = 0$, and $C = 7$, $Ax + By + C = 0$ becomes $3x + 0y + 7 = 0$ or $3x + 7 = 0$. This, too, is an acceptable equation for a line.

If $A = 0$, $B = 0$, and $C = 7$, $Ax + By + C = 0$ becomes $0x + 0y + 7 = 0$ or $7 = 0$. This is not true.

In Mathematics 9 you sketched linear equations using a **table of values**. To graph the linear equation $3x + y + 6 = 0$, you rewrite the equation as follows:

$$\begin{aligned} 3x - 3x + y + 6 &= 0 - 3x \\ y + 6 &= -3x \\ y + 6 - 6 &= -3x - 6 \\ y &= -3x - 6 \end{aligned}$$

When you have written the equation as

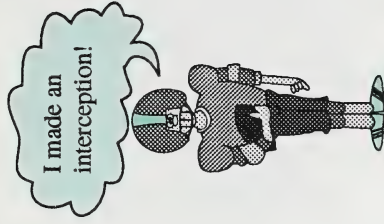
$y = -3x - 6$, you have made y the subject of the equation.

You now have a form of the equation to find values for y when you are given values for x .

Suppose x is replaced by -3 .

$$\begin{aligned} y &= -3x - 6 \\ &= -3(-3) - 6 \\ &= 9 - 6 \\ &= 3 \end{aligned}$$

Therefore, when $x = -3$, $y = 3$.



What have you just found?

You have found a point on the graph of the equation. The point is $(-3, 3)$.

Remember that when you write an ordered pair, x is written first as shown in (x, y) .

Now you can find other points by completing the following table of values.

$$y = -3x - 6$$

x	-3	-2	-1	0	1	2	3
y	3						

When x is replaced by -2 , y is calculated as follows:

$$\begin{aligned} y &= -3(-2) - 6 \\ &= 6 - 6 \\ &= 0 \end{aligned}$$

When x is replaced by -1 , y is calculated as follows:

$$\begin{aligned} y &= -3(-1) - 6 \\ &= 3 - 6 \\ &= -3 \end{aligned}$$

When x is replaced by 0 , y is calculated as follows:

$$\begin{aligned} y &= -3(0) - 6 \\ &= 0 - 6 \\ &= -6 \end{aligned}$$

When x is replaced by 1 , y is calculated as follows:

$$\begin{aligned} y &= -3(1) - 6 \\ &= -3 - 6 \\ &= -9 \end{aligned}$$

When x is replaced by 2 , y is calculated as follows:

$$\begin{aligned} y &= -3(2) - 6 \\ &= -6 - 6 \\ &= -12 \end{aligned}$$

When x is replaced by 3 , y is calculated as follows:

$$\begin{aligned} y &= -3(3) - 6 \\ &= -9 - 6 \\ &= -15 \end{aligned}$$

The choice of x -values is up to you. The integers $-3, -2, -1, 0, 1, 2$, and 3 were chosen to obtain points on both sides of the y -axis. Many other values could have been used as x -values.

The complete table of values is as follows.

x	-3	-2	-1	0	1	2	3
y	3	0	-3	-6	-9	-12	-15

These are the ordered pairs.

$(-3, 3), (-2, 0), (-1, -3), (0, -6), (1, -9), (2, -12), (3, -15)$

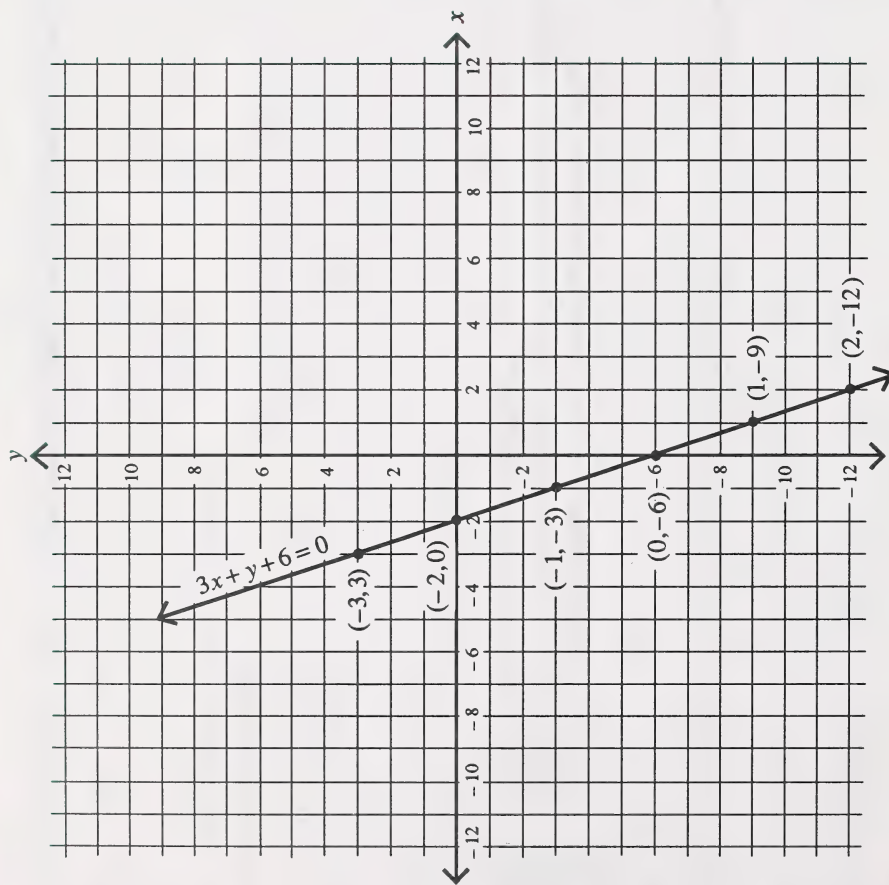
Plot these points and then join them using a straightedge. A line results when the points are joined. This line is called the graph of the equation. Next, you will find the graph of the equation $3x + y + 6 = 0$.



If you have a graphing calculator or a graphing computer program available, you may use it to check the equations graphed in this topic. Check with the manual for instructions on how to use your graphing aid.

You do not have to plot all of the points you found in the table of values in order to graph the line. You only need two points to draw the graph of the line.





The graph of the equation $3x + y + 6 = 0$ is a straight line. In fact, the graph of any linear equation of the form $Ax + By + C = 0$ is a straight line.

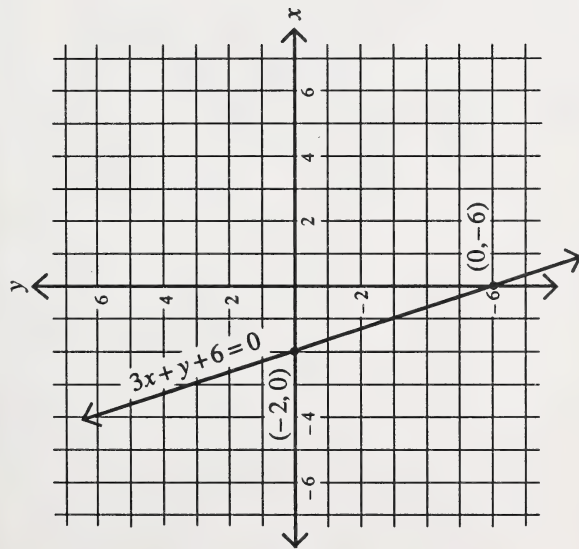


Did you notice that the points lie on the same straight line? These are collinear points. The word **collinear** contains the word **line**.



The expression $Ax + By + C = 0$ is referred to as the standard form of the equation of a straight line.

There is a faster way to graph linear equations. The point $(-2, 0)$ lies on the x -axis since the y -coordinate is 0. The point $(0, -6)$ lies on the y -axis since the x -coordinate is 0. Examine the points where the graph of $3x + y + 6 = 0$ crosses the axes.



The graph crosses the x -axis at $(-2, 0)$.

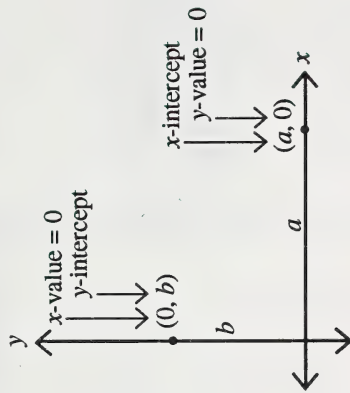
The x -intercept is -2 .

The line crosses the y -axis at $(0, -6)$.

The y -intercept is -6 .



In general, the following applies.



Before you are given more facts about the equation of a straight line, you should look at a few more examples.

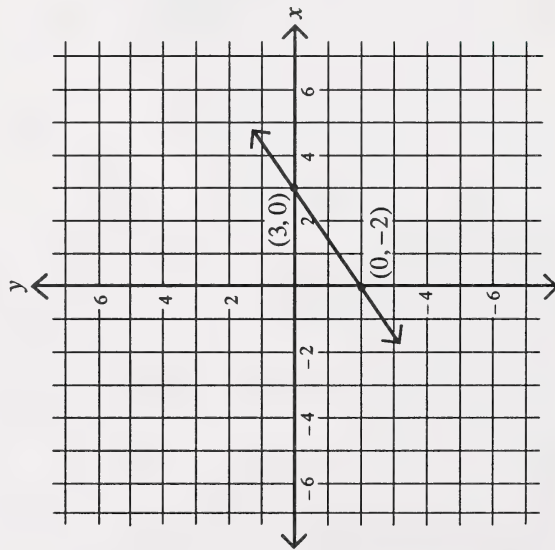


Example 1

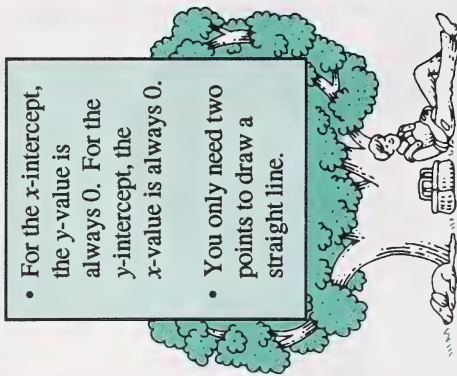
Draw the line with an x -intercept of 3 and a y -intercept of -2 .

Solution:

The x -intercept is 3. The line crosses the x -axis at $(3, 0)$. The y -intercept is -2 . The line crosses the y -axis at $(0, -2)$.



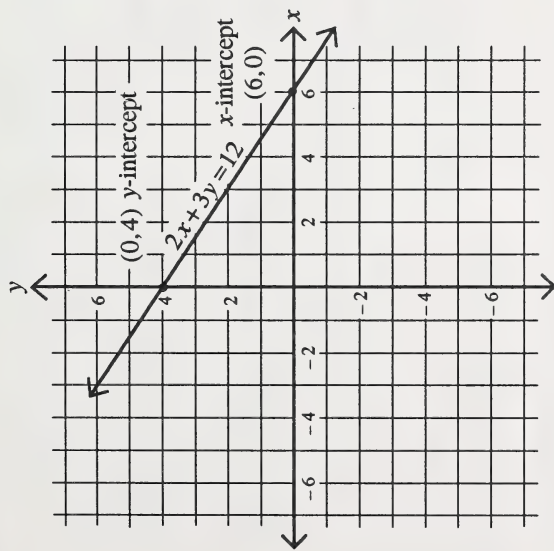
The intercepts provide a quick method for drawing a linear equation.



Example 2

Graph $2x + 3y = 12$ using its intercepts.

Solution:



To find the x-intercept, let $y = 0$.

$$2x + 3y = 12$$

$$2x + 3(0) = 12$$

$$2x = 12$$

$$x = 6$$

The graph crosses the x-axis at $(6, 0)$.

To find the y-intercept, let $x = 0$.

$$2x + 3y = 12$$

$$2(0) + 3y = 12$$

$$3y = 12$$

$$y = 4$$

The graph crosses the y-axis at $(0, 4)$.

If the x- and y-intercepts are both zero, you will need additional information to draw the graph. Another point on the line would provide that information.

You will see this in the next example.

You should always write the equation of the line along the graph of the line.

Example 3

Draw the graph for $2x + y = 0$.

Solution:

To find the x -intercept, let $y = 0$.

$$2x + y = 0$$

$$2x + 0 = 0$$

$$2x = 0$$

$$x = 0$$

To find the y -intercept, let $x = 0$.

$$2x + y = 0$$

$$2(0) + y = 0$$

$$y = 0$$

Both intercepts equal 0.

The line must pass through the origin which is the point $(0, 0)$.

Because you need two points to sketch a line, you should complete a partial table of values.

When $x = 1$, y is as follows:

$$2x + y = 0$$

$$2(1) + y = 0$$

$$2 + y = 0$$

$$y = -2$$

One ordered pair on the graph is $(1, -2)$.

When $x = 2$, y is as follows:

$$2x + y = 0$$

$$2(2) + y = 0$$

$$4 + y = 0$$

$$y = -4$$

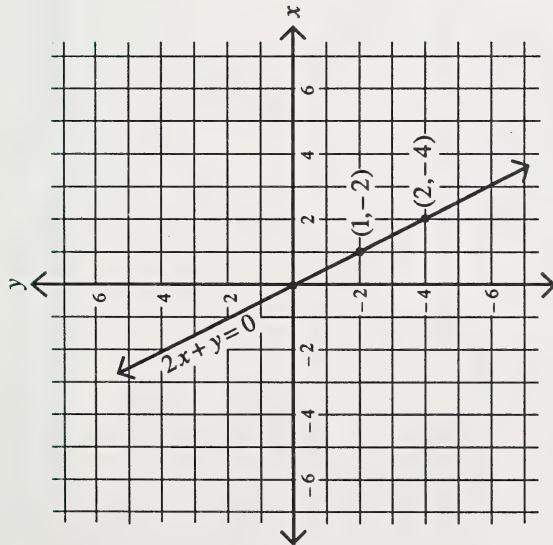
Another ordered pair on the graph is $(2, -4)$.

x	0	1	2
y	0	-2	-4

The following is the graph of $2x + y = 0$.

You choose your own x -values for additional points on the graph. In this case, 1 and 2 were chosen. You could have used any value you wanted. Your graph would still be the same.





Now consider lines parallel to the axes. A line parallel to the x -axis takes the form $y = b$, where b is a real number representing the distance from the x -axis to the line.

A line parallel to the y -axis takes the form $x = a$, where a is a real number representing the distance from the y -axis to the line.

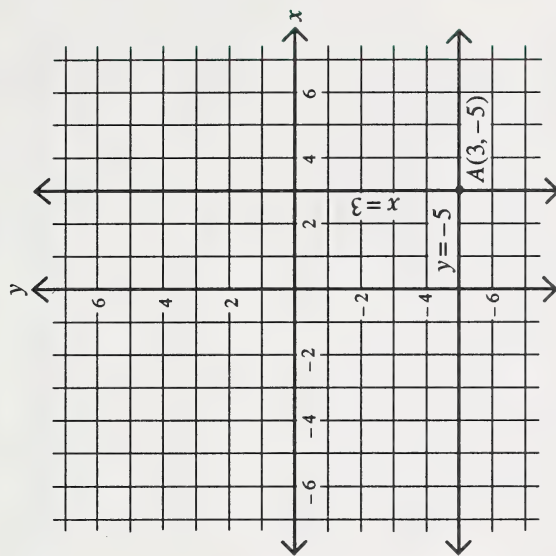
You should understand this better once you have looked at the next example.

Example 4

What vertical and horizontal lines pass through the point $A(3, -5)$? Write their equations.

Solution:

Make a graph of the vertical line and the horizontal line passing through $A(3, -5)$.

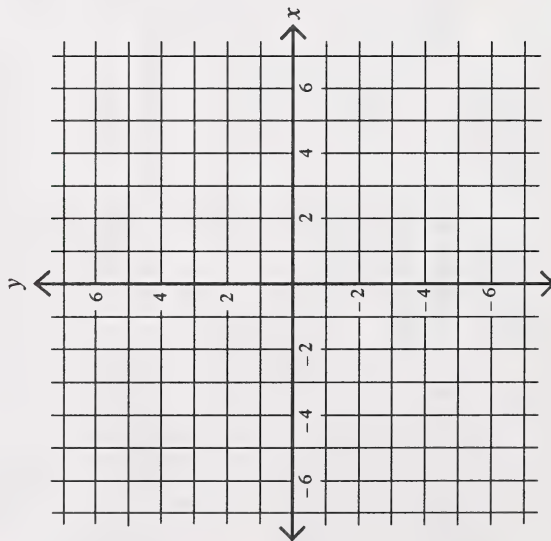


At the point $A(3, -5)$, x has a value of 3 and y has a value of -5 . Therefore, $x = 3$ and $y = -5$. The equation $x = 3$ can be written in standard form as $x - 3 = 0$. The equation $y = -5$ can be written in standard form as $y + 5 = 0$. Thus, $x - 3 = 0$ and $y + 5 = 0$ are the equations of the vertical and horizontal lines passing through $A(3, -5)$.

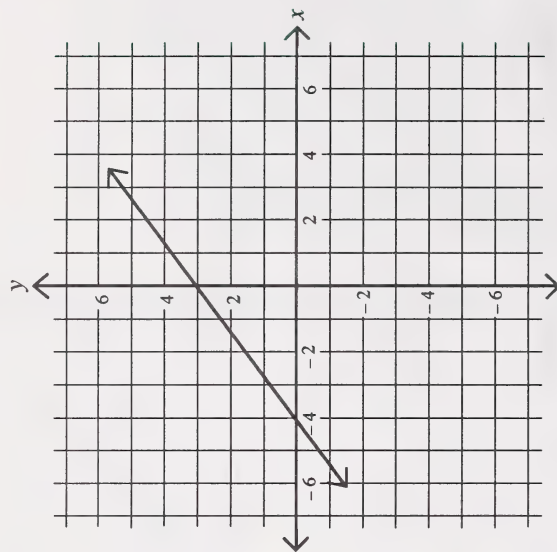
Complete any five of the following questions.

1. Draw the graph of the line represented by the equation $2x - y - 6 = 0$. Complete the table of values first.

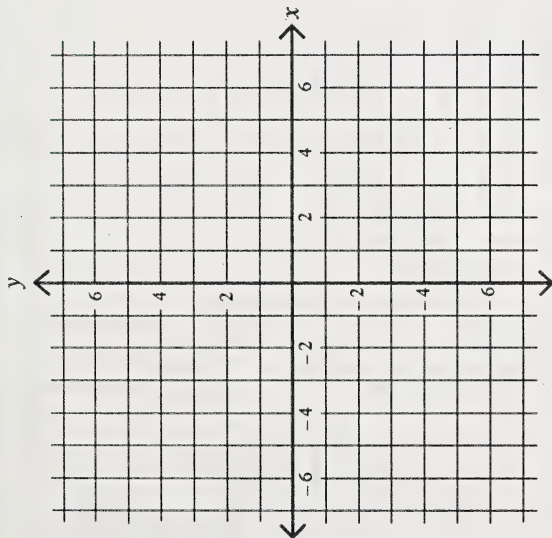
x	-2	-1	0	1	2	3
y						



2. Find the intercepts of the line graphed as follows. What is its slope?



3. Use any method to graph the line $x - 3y = 0$.

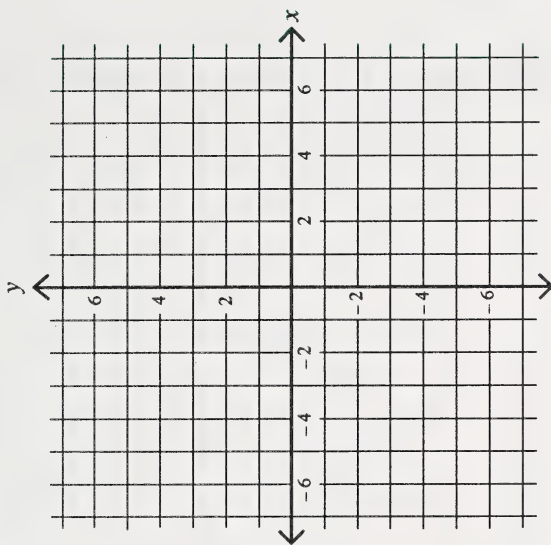


4. a. Find the x - and y -intercepts of the line $3x - 4y - 12 = 0$.

b. At what point does the line intersect the x -axis?

c. At what point does the line intersect the y -axis?

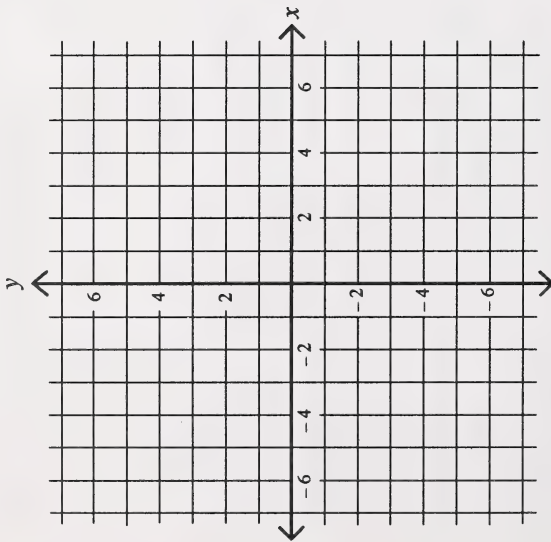
d. Use the x -intercept and the y -intercept to graph the equation.



5. a. Draw the graphs of the lines $2x + 3y - 6 = 0$ and $4x + 6y = -20$.

b. Do they meet?

c. State the x - and y -intercepts for both lines.



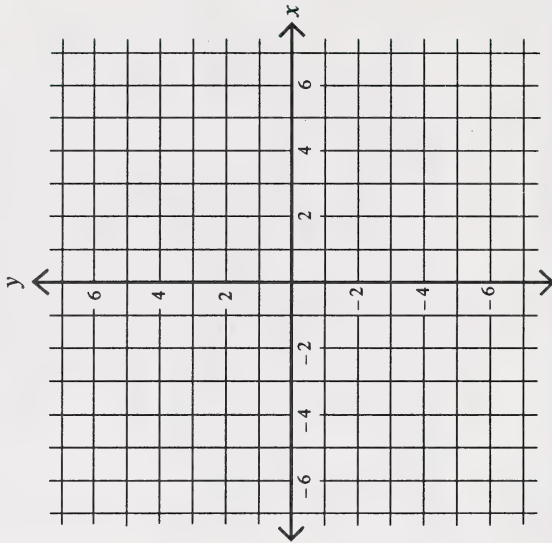
6. Complete the following table of values for the equation $3x + 2y - 12 = 0$.

x	-4	-2	0	2	4
y					

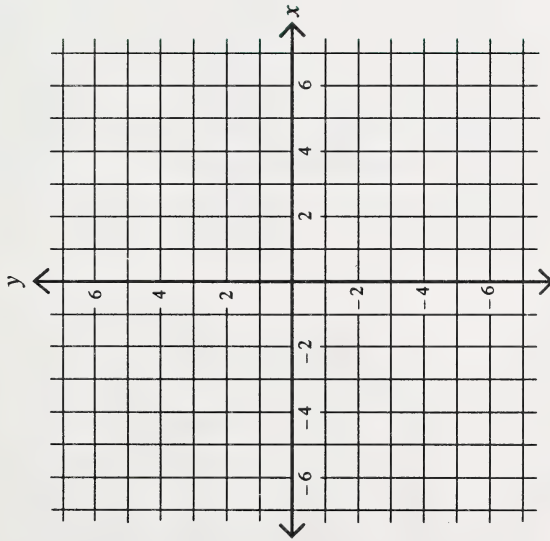
7. a. Graph the line defined by the following table of values.

x	-3	-2	-1	0	1
y	3	0	-3	-6	-9

- b. Find the x - and y -intercepts.
c. At what points does the line cut the x - and y -axes?

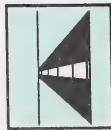


8. a. Draw the line with an x-intercept of -4 and a y-intercept of 6 .
b. What is the slope of this line?



For solutions to Activity 1, turn to the Appendix, Topic 3.

Activity 2



Write and graph the equation of a line in the form $y = mx + b$.

Earlier you learned about sketching a line on the coordinate system by using either a table of values or the x - and y -intercepts. You also learned how to find the slope of a line either by counting squares or by calculating rise over run.

You will now go one step further in studying the linear equation.

You have studied the standard form of an equation. It is $Ax + By + C = 0$, where both A and B cannot equal zero. You can write this equation in another form. This form is called the **slope y -intercept form**.

Consider $y - 2x - 6 = 0$.

Now rewrite this equation expressing y in terms of x .

$$y - 2x - 6 = 0$$

$$y - 2x - 6 + 6 = 0 + 6$$

$$y - 2x = 6$$

$$y - 2x + 2x = 6 + 2x$$

$$y = 2x + 6$$

To draw the graph of this equation, you calculate the x - and y -intercepts.

To find the x -intercept, let $y = 0$.

$$y = 2x + 6$$

$$0 = 2x + 6$$

$$0 - 6 = 2x + 6 - 6$$

$$-6 = 2x$$

$$x = \frac{-6}{2}$$

$$= -3$$

The x -intercept is -3 .

To find the y -intercept, let $x = 0$.

$$y = 2x + 6$$

$$= 2(0) + 6$$

$$= 0 + 6$$

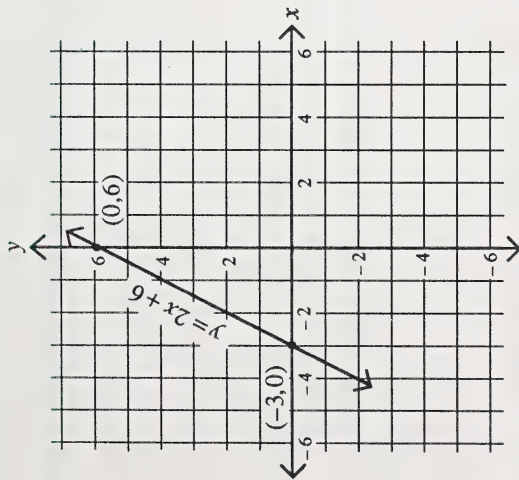
$$= 6$$

The y -intercept is 6 .

The standard form of a linear equation is $Ax + By + C = 0$, where A and B cannot both be zero.



The slope y -intercept form of an equation is $y = mx + b$, where m is the slope and b is the y -intercept.



Let $(-3, 0) = (x_1, y_1)$ and $(0, 6) = (x_2, y_2)$.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\therefore m = \frac{6 - 0}{0 - (-3)}$$

$$= \frac{6}{3}$$

$$= 2$$

Notice that the slope appears in the equation as the **coefficient of the variable term**.

$$y = 2x + 6$$

↑
slope

Notice that the y-intercept appears in the equation as the constant term.

$$y = 2x + 6$$

↑
y-intercept

Now the slope of the line can be determined using the following formula.

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$



Note that $y = mx + b$ is called the **slope y-intercept form** of a linear equation.

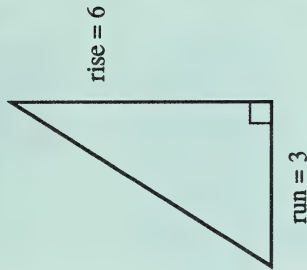
$$y = mx + b \leftarrow \begin{matrix} \uparrow \\ \text{slope} \end{matrix} \begin{matrix} \leftarrow \\ \text{y-intercept} \end{matrix}$$



$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{6}{3}$$

$$= 2$$



Another way to find the slope would be to count the squares for the rise and the run in a triangle such as this and then to calculate as shown.

Compare $y = 2x + 6$ with $y = mx + b$.

$$\begin{aligned}\text{Slope} &= m \\ &= 2 \\ \text{y-intercept} &= b \\ &= 6\end{aligned}$$

You can now use the **slope y-intercept form** to draw the graph of a linear equation.

Example 5

Draw the graph of $y = 3x - 4$ using the slope and y-intercept.

Solution:

Compare $y = 3x - 4$ with $y = mx + b$.

$$\begin{aligned}m &= 3 \\ b &= -4\end{aligned}$$

Therefore, the slope is 3 and the y-intercept is -4 .

Because the y-intercept is -4 , the graph crosses the y-axis at $(0, -4)$. Mark $(0, -4)$ on the graph. You can use the slope to locate a second point on the line.

Use the formula for slope:

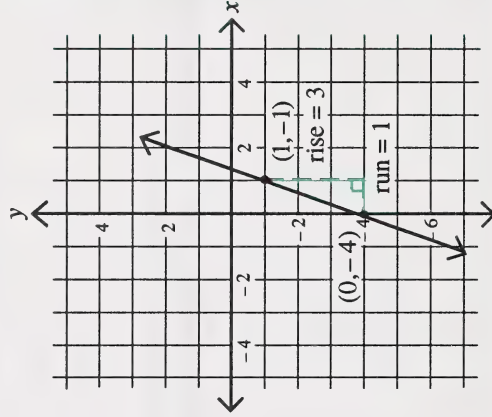
$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ \frac{3}{1} &= \frac{\text{rise}}{\text{run}}\end{aligned}$$

Remember that $3 = \frac{3}{1}$.

Using this relationship, the rise is 1 and the run is 3.

Start at the point $(0, -4)$ and go one unit to the right and three units up. The coordinates of the second point are $(1, -1)$.

Now draw a line through $(0, -4)$ and $(1, -1)$. Look at the following graph.



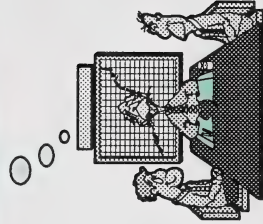
When calculating slope, you can count off the spaces for the rise and the run and express this as a fraction.

$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{1} \text{ or } 3\end{aligned}$$



The letter b is used as the symbol for the y-intercept.

The y-intercept is the place where the graph crosses the y-axis.



Notice that in Example 5 you were given the equation of the line in slope y-intercept form. This is not always the case. If you are given the equation of a line in standard form, you should be able to change it to slope y-intercept form.

The next example will demonstrate this.

Example 6

Draw the graph of $x + 4y = 8$ using its slope and y-intercept.

Solution:

You must begin by writing the equation in the form $y = mx + b$.

$$x + 4y = 8$$

$$-x + x + 4y = -x + 8$$

$$4y = -x + 8$$

$$\frac{4y}{4} = \frac{-x + 8}{4}$$

$$y = -\frac{1}{4}x + 2$$

Since $y = mx + b$, $m = -\frac{1}{4}$ and $b = 2$.

The y-intercept is 2. Therefore, $(0, 2)$ is one point. Now you need to find the coordinates of a second point.

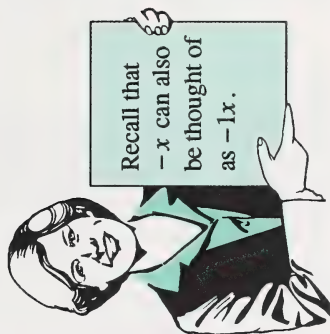
Use the formula for slope.

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

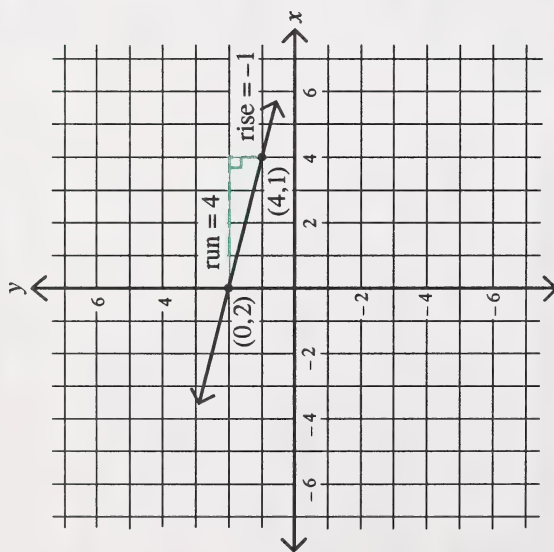
$$m = \frac{\text{rise}}{\text{run}}$$

$$-\frac{1}{4} = \frac{\text{rise}}{\text{run}}$$

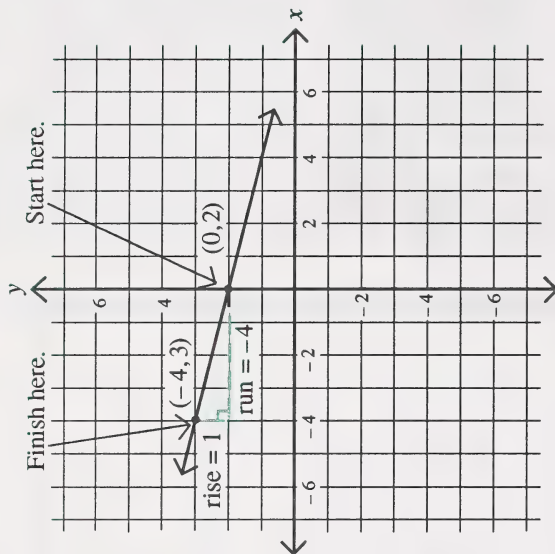
Therefore, the run is 4 and the rise is -1 .



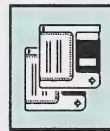
To find the second point, start at $(0, 2)$ and go four units to the right and one unit down. You arrive at $(4, 1)$. Now draw a line through the points to draw the graph of the equation. You should end up with a graph like this.



If you had used -4 as the run and $+1$ as the rise, the graph would still look the same.



Now you may like to try this.



Go to the computer program entitled *Algebra Drill and Practice I*,¹ Part 3, Section 2 on finding slope given the equation. Follow some of the examples first if necessary, and then do as many of the questions as you wish.

¹ *Algebra Drill and Practice I* is a title of Conduit.

A quick way of moving a term to the other side of an equation is to change the sign and add it to the other side. For example, in $3x + y - 6 = 0$, change $3x$ to $-3x$ and change -6 to 6 and then add $-3x$ and 6 to 0 to get $y = -3x + 6$.

Take another example such as $4x + 5y = 10$. Change $4x$ to $-4x$ and add it to the 10 . You get $5y = -4x + 10$. Then divide all the terms by the coefficient of y which in this case is 5 . You end up with the following:

$$\frac{5y}{5} = \frac{-4x}{5} + \frac{10}{5}$$

When simplified, this becomes $y = -\frac{4}{5}x + 2$.

Do any five of the following seven questions.

For questions 1 to 5, complete the following.

- Write the equation in the form $y = mx + b$.
- Give the values of m and b .
- Give the x -intercept.

(Hint: You might like to make a table to record your answers.)

1. $2x + y - 7 = 0$
2. $3y = 2x - 9$
3. $4x + 5y = 20$
4. $x - 3y = -9$
5. $y - 4 = 0$

6. Write each of the following in the standard form

$$Ax + By + C = 0.$$

a. $y = 6 - 2x$

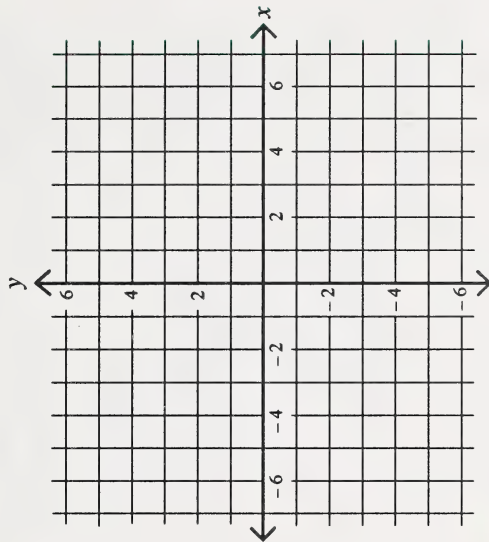
b. $2y + 3x = \frac{1}{2}$

c. $\frac{x-2}{3} = y$

d. $y = mx + b$

e. $\frac{x}{2} - \frac{y}{3} + 1 = 0$

7. Draw the graph for the line $y = 3x + 2$ using the slope and y -intercept.



For solutions to Activity 2, turn to the Appendix, Topic 3.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

} You may decide to do both.



Extra Help

Lines on a coordinate plane may run in one of three ways.

- parallel to the x -axis
- parallel to the y -axis
- slanted to either of the axes (obliquely)

The equation for each of these takes a certain form.

- The equation $Ax + C = 0$ indicates the line is parallel to the y -axis.
- The equation $By + C = 0$ indicates the line is parallel to the x -axis.
- The equation $Ax + By + C = 0$ indicates an oblique line.

Example 7

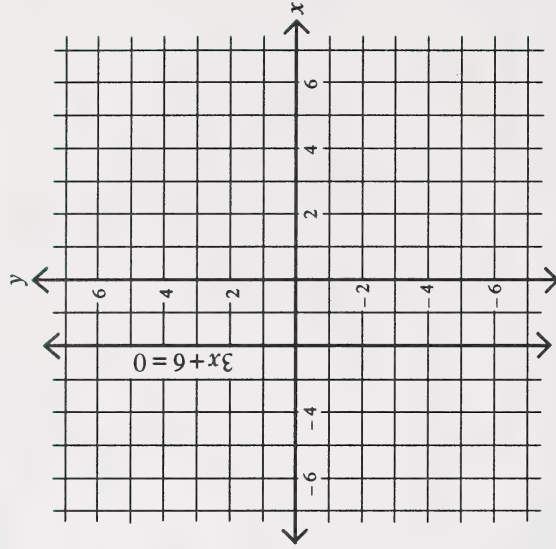
- Draw the graph of $3x + 6 = 0$.

Solution:

$$3x + 6 = 0$$

$$3x = -6$$

$$x = -2$$



For every point on the line $x = -2$, the first coordinate would be -2 .

- Draw the graph of $4y - 8 = 0$.

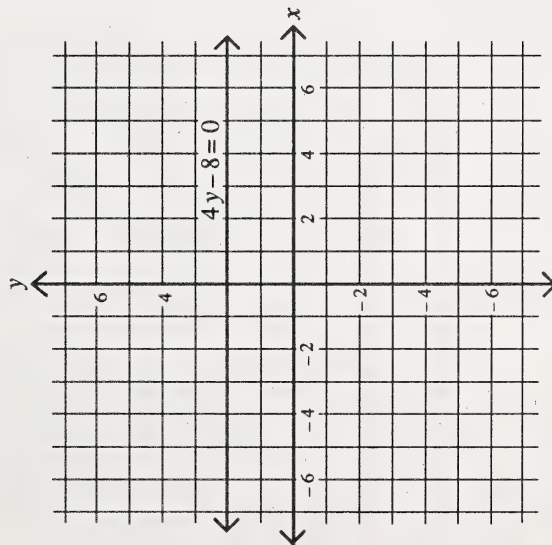
Solution:

$$4y - 8 = 0$$

$$4y = 8$$

$$y = \frac{8}{4}$$

$$= 2$$



For every point on the line $y = 2$, the second coordinate would be 2.

- Draw the graph of $x + y - 4 = 0$.

Solution:

$$x + y - 4 = 0$$

$$y = -x + 4$$

To find the y-intercept,

$$\text{let } x = 0.$$

$$x + y - 4 = 0$$

$$0 + y - 4 = 0$$

$$y - 4 = 0$$

$$y = 4$$

To find the x-intercept,

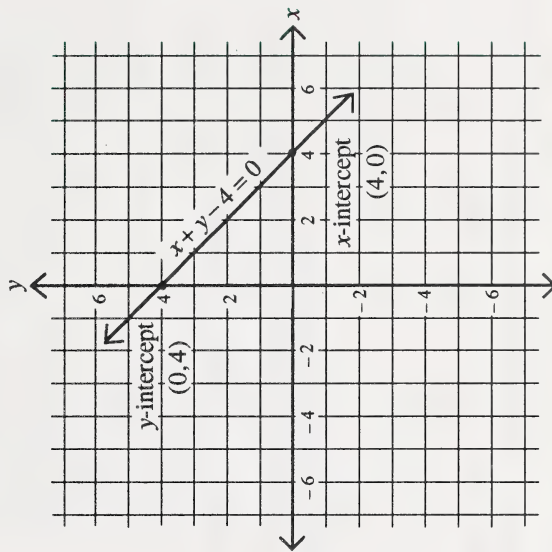
$$\text{let } y = 0.$$

$$x + y - 4 = 0$$

$$x + 0 - 4 = 0$$

$$x - 4 = 0$$

$$x = 4$$



If you are given **two points** on a coordinate plane, you should be able to draw the graph. If you are given **one point** and the **slope** of a line, you should be able to draw the graph. As you become more familiar with this subject, you will be able to tell just how much information you need in order to draw a line.

Look at this example.

Example 8

A line passes through the points $A(0, -3)$ and $B(3, 0)$.

- What is the slope of the line joining A and B ?

Solution:

$$\begin{aligned}\text{Slope}(m) &= \frac{\text{rise}}{\text{run}} \\ &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{0 - (-3)}{3 - 0} \\ &= \frac{3}{3} \\ &= 1\end{aligned}$$

The slope of the line is 1.

- What is the equation of the line joining A and B ?

Solution:

Point $A(0, -3)$ gives you the y -intercept which is -3 .

Substitute $m = 1$ and $b = -3$ into $y = mx + b$.

$$y = 1(x) + (-3)$$

$$y = x - 3$$

This is the slope y -intercept form.

In standard form, the equation of the line is $x - y - 3 = 0$.

Note how you can get $x - y - 3 = 0$ from $y = x - 3$.

$$y = x - 3$$

$$y - x = x - x - 3$$

$$y - x = -3$$

$$y - x + 3 = -3 + 3$$

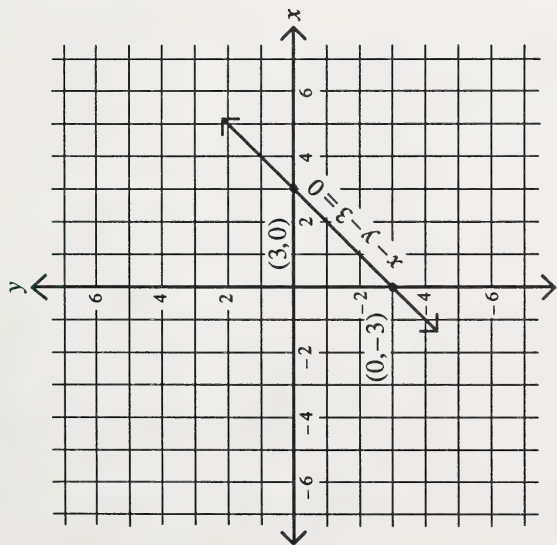
$$y - x + 3 = 0$$

$$-x + y + 3 = 0$$

Multiply every term by -1 to get $x - y - 3 = 0$.

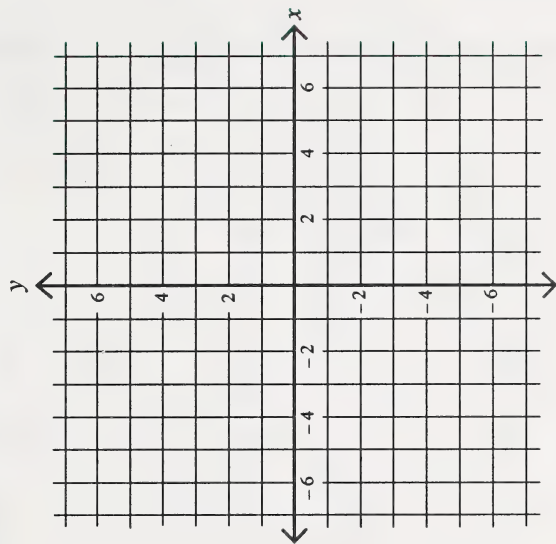
- Draw the graph of the equation.

Solution:



Try some of the following questions.

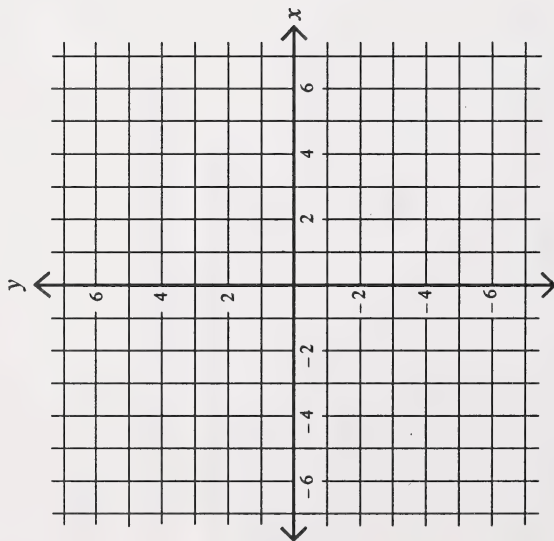
1. a. Find the slope of the line given by $2x - 3y - 7 = 0$.
b. What is the y-intercept of this line?
2. What are the x- and y-intercepts of the line given by $2x + 3y - 12 = 0$?
3. Draw the graph of $4x - 3y - 12 = 0$.



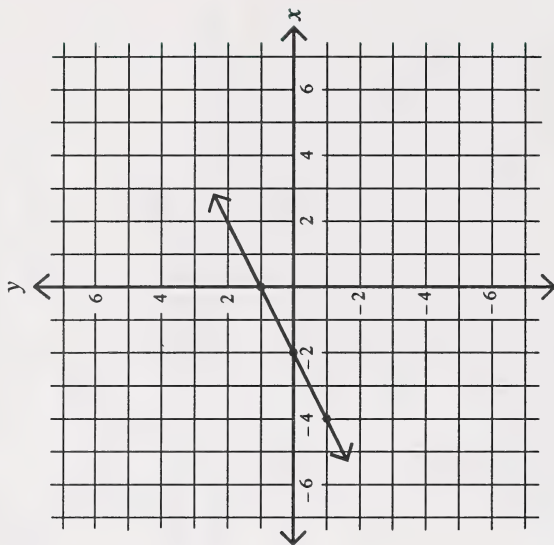
4. Graph the following equations on the same coordinate plane.

$$2x - 6 = 0$$

$$3y + 15 = 0$$



5. Write the equation of the line on this coordinate plane. (Hint: Use the coordinate plane to determine two points on the line. What is the y-intercept? What is the x-intercept?)



For solutions to **Extra Help**, turn to the **Appendix, Topic 3**.



Extensions

Now that you are able to change an equation from standard form to slope y-intercept form, you can go one step further. You can find the slope of a line that is perpendicular to a given line.

Before you do this, you should recall the following:

- Parallel lines have the same slope.
- The product of the slopes of perpendicular lines equals -1 .

Now study this example.

Example 9

Find the slope of the line perpendicular to $3x + 2y = 8$.

Solution:

First find the slope of $3x + 2y = 8$ by writing it in slope y-intercept form.

$$3x + 2y = 8$$

$$2y = -3x + 8$$

$$\frac{2y}{2} = \frac{-3x}{2} + \frac{8}{2}$$

$$y = -\frac{3}{2}x + 4$$

Since $y = mx + b$, $m = -\frac{3}{2}$.

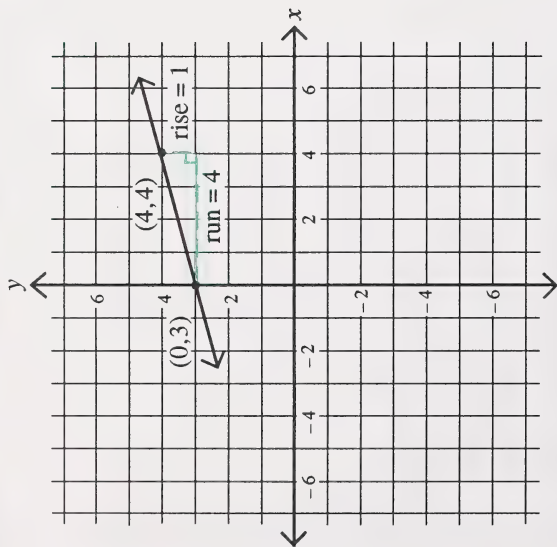
The slope of $3x + 2y = 8$ is $-\frac{3}{2}$.

The slope of the line perpendicular to the line formed by $3x + 2y = 8$ would be the negative reciprocal of $-\frac{3}{2}$ which is $\frac{2}{3}$.

Every line on a graph is defined by an equation. In the next example a line is drawn and you are asked to find the equation of that line.

Example 10

Write the equation of the line drawn as follows.



Solution:

Find the slope of the line.

$$\begin{aligned} m &= \frac{\text{rise}}{\text{run}} \\ &= \frac{1}{4} \end{aligned}$$

The y-intercept is 3. Therefore, $b = 3$.

Using $y = mx + b$, the equation of the line is $y = \frac{1}{4}x + 3$.

Did you know that $y = \frac{1}{4}x + 3$ can be written in standard form as follows?

$$y = \frac{1}{4}x + 3$$

$$4(y) = 4\left(\frac{1}{4}x\right) + 4(3)$$

$$4y = x + 12$$

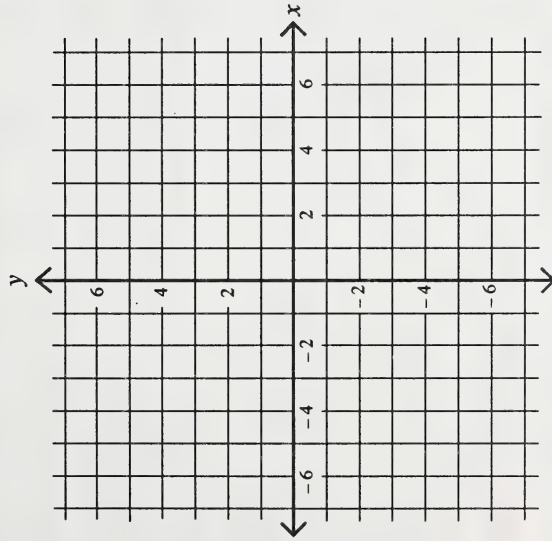
$$x - 4y + 12 = 0$$



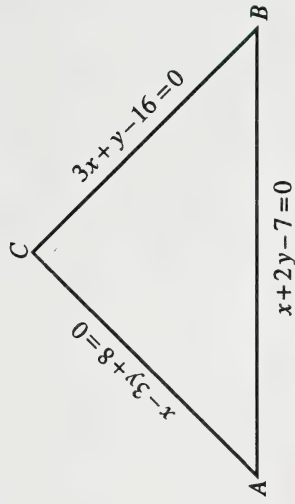
Try some of the questions that follow.

1. a. Find the slope of a line parallel to the line $3x - 2y = 6$.
- b. Find the slope of a line perpendicular to the line $3x - 2y - 6 = 0$.

2. Write the equation of the line that passes through the points $A(0, 1)$ and $B(4, 4)$. Draw the graph of the line.



3. Triangle ABC is formed by the intersection of lines $x - 3y + 8 = 0$, $x + 2y - 7 = 0$, and $3x + y - 16 = 0$. Determine whether or not $\triangle ABC$ is a right triangle. Give reasons for your answer.



For solutions to Extensions, turn to the Appendix, Topic 3.

Topic 4 Distance Between Two Points



Introduction

A bush plane went down in the forests of British Columbia. It was spotted on Lyell Island by a ranger who radioed the nearest city for help. A rescue team from Prince Rupert responded immediately. By using map coordinates, compasses, and other tracking devices, the rescue team was able to locate the downed plane.

You can use the coordinates of two points in a Cartesian plane to find the distance between them. In this topic you will use the Pythagorean theorem to develop the distance formula.



What Lies Ahead

Throughout the topic you will learn to

1. use the Pythagorean theorem and the distance formula to find the distance between two points
2. use the formula $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$ to find the midpoint of a line segment

Now that you know what to expect, turn the page to begin your study of distance between two points.



Exploring Topic 4

Activity 1



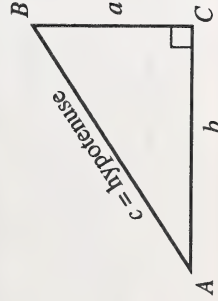
Use the Pythagorean theorem and the distance formula to find the distance between two points.

Many years ago a Greek philosopher discovered that he could calculate distances without having to make the actual measurements. This great thinker was Pythagoras. He discovered an interesting relation among the sides of a right triangle. A right triangle always has one side that is longer than the other two sides. This side is called the hypotenuse.



Pythagoras found that the square on the hypotenuse is equal to the sum of the squares on the other two sides.

This relation is referred to as the Pythagorean theorem. This diagram shows the theorem.



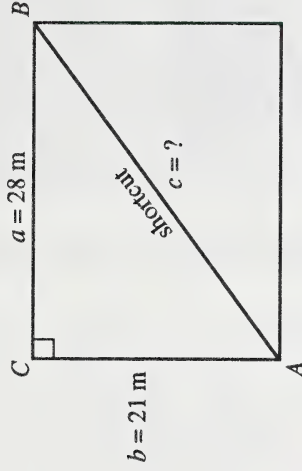
Pythagorean theorem:

$$c^2 = a^2 + b^2$$

The following example shows how the Pythagorean theorem can be applied to an everyday situation.

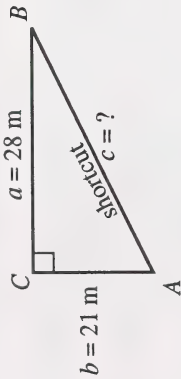
Example 1

A Mathematics 13 student decides to take a shortcut across the vacant lot shown in the diagram. What distance does the student save by taking the shortcut instead of walking around the lot from A to C and then to B?



Solution:

The hypotenuse in the diagram is c .



$$c^2 = a^2 + b^2$$

$$c^2 = 28^2 + 21^2$$

$$c^2 = 784 + 441$$

$$c^2 = 1225$$

$$\sqrt{c^2} = \pm\sqrt{1225}$$

$$c = \pm 35$$

The shortcut is 35 m in length.

The -35 value for c is ignored because a length cannot be negative.

If the student does not take the shortcut, $21\text{ m} + 28\text{ m}$ or 49 m would be walked. Therefore, the student saves $49\text{ m} - 35\text{ m}$ or 14 m by taking the shortcut.

On many occasions you will find that the hypotenuse is not a natural number. When this is the case, you will need to use your calculator. Here is one such example.

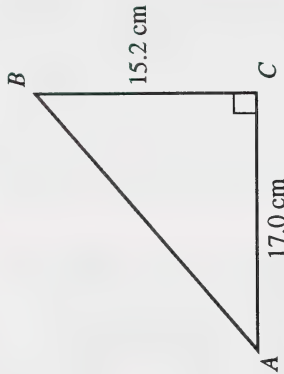
Example 2

Use a calculator to find the hypotenuse of a right triangle if the two other sides are 17 cm and 15.2 cm . Round to one decimal place.

Solution:

By the Pythagorean theorem,

$$(AB)^2 = (AC)^2 + (BC)^2.$$



Natural numbers are $1, 2, 3, \dots$. The three dots mean that the numbers continue on to infinity.





Use a calculator as follows:

Enter	Display
17.0	17.0
x^2	289
+	289
15.2	15.2
x^2	231.04
=	520.04
$\sqrt{}$	± 22.80438554

The hypotenuse of the triangle is 22.8 cm correct to one decimal place.

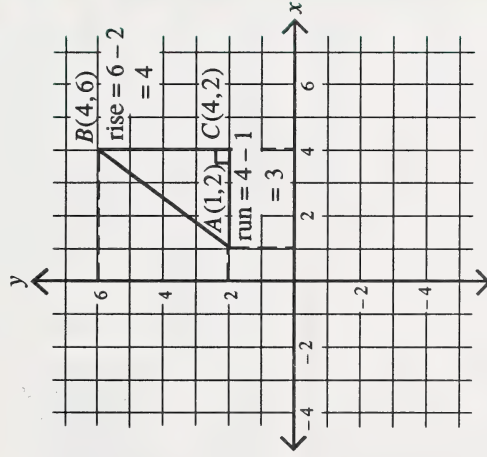
Now that you have an understanding of the Pythagorean theorem, you can apply this knowledge to the coordinate plane. You can find the distance between two points if the coordinates of the points are given. Look at this example to see how this works.

Example 3

Calculate the distance between $A(1, 2)$ and $B(4, 6)$.

Solution:

Draw a diagram. Plot the points. Calculate the rise and the run.



Note the following:

- For some calculators you have to enter the 2^{nd} F key before the $\sqrt{}$ key.
- Notice that $\sqrt{520.04}$ is between ± 22 and ± 23 since $22^2 = 484$ or $(-22)^2 = 484$ and $23^2 = 529$ or $(-23)^2 = 529$.
- The negative square roots are ignored since a measure cannot be negative.

Through calculations you can find $(AC)^2$ and $(BC)^2$.

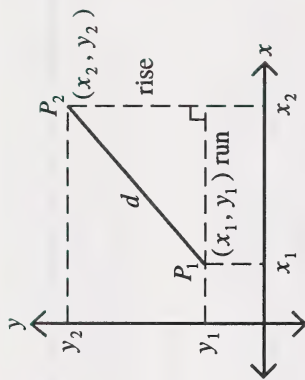
$$\begin{aligned} \text{run} &= AC \\ &= 4 - 1 \\ &= 3 \\ \text{rise} &= BC \\ &= 6 - 2 \\ &= 4 \\ \therefore (AC)^2 &= 3^2 \\ &= 9 \\ (BC)^2 &= 4^2 \\ &= 16 \end{aligned}$$

From the Pythagorean theorem, you get the following:

$$\begin{aligned} (AB)^2 &= (AC)^2 + (BC)^2 \\ (AB)^2 &= 9 + 16 \\ (AB)^2 &= 25 \\ \sqrt{(AB)^2} &= \pm\sqrt{25} \\ AB &= 5 \text{ units} \end{aligned}$$

The negative square root is ignored.

You can now apply the results of Example 3 to develop a formula for the distance between any two points in the coordinate plane.



$$\begin{aligned} P_1 &= (x_1, y_1) \\ P_2 &= (x_2, y_2) \end{aligned}$$

Let d be the distance between points P_1 and P_2 .

In the diagram, run = $x_2 - x_1$ and rise = $y_2 - y_1$.

Therefore,

$$\begin{aligned} d^2 &= (\text{run})^2 + (\text{rise})^2 \\ d^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 \\ d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \end{aligned}$$



This is the **distance formula**. It is used to find the distance between any two points on a coordinate plane.

The symbol AB represents the measure of the distance from point A to point B.



To find d from

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$, take the square root of both sides of the equation to get the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Be an angel and memorize the formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



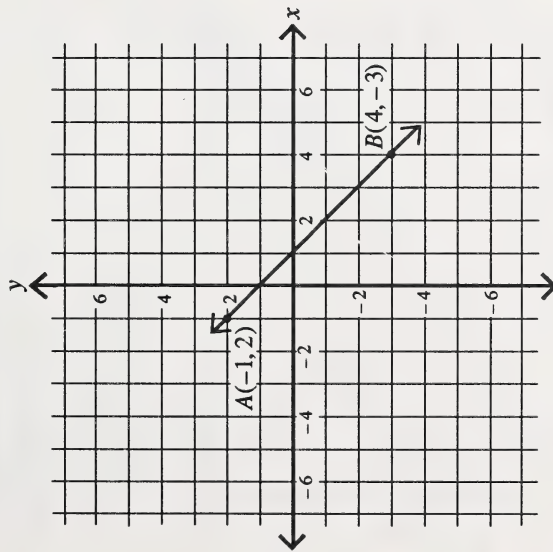
In the next example you will see how to calculate the distance between two points in the coordinate plane.

Example 4

Calculate the distance between $A(-1, 2)$ and $B(4, -3)$.

Solution:

Plot the points and join the points with a line.



Let $A(-1, 2) = (x_1, y_1)$.

Let $B(4, -3) = (x_2, y_2)$.

Use the formula for distance.

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ AB &= \sqrt{[4 - (-1)]^2 + (-3 - 2)^2} \\ &= \sqrt{(4+1)^2 + (-5)^2} \\ &= \sqrt{5^2 + 25} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \end{aligned}$$

Using a calculator, you will find that the distance between A and B is 7.1 when rounded to one decimal place.

You could have reversed the order of the points.

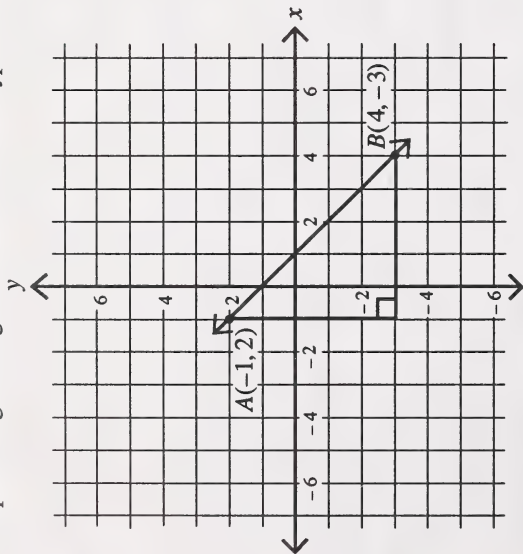
If $B(4, -3) = (x_1, y_1)$ and $A(-1, 2) = (x_2, y_2)$, then AB would be calculated as follows:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ AB &= \sqrt{(-1 - 4)^2 + [2 - (-3)]^2} \\ &= \sqrt{(-5)^2 + (2+3)^2} \\ &= \sqrt{25 + 25} \\ &= \sqrt{50} \\ &\approx 7.1 \end{aligned}$$

The order in which the points are used does not change the final answer.

Check:

Complete a right triangle with side AB as the hypotenuse.



rise = 5 when spaces are counted
run = 5 when spaces are counted

$$c^2 = a^2 + b^2$$

$$c^2 = 5^2 + 5^2$$

$$c^2 = 25 + 25$$

$$c^2 = 50$$

$$\sqrt{c^2} = \sqrt{50}$$

$$c \doteq 7.071\ 067\ 812$$

$$\doteq 7.1 \quad (\text{rounded to one decimal place})$$

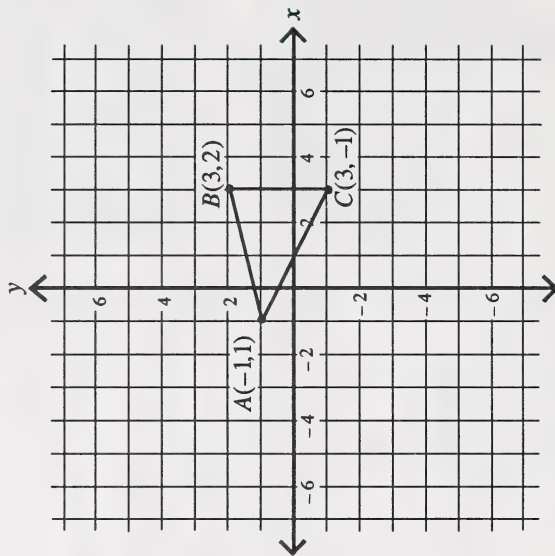
Now study an example in which you are asked to find the perimeter of a triangle in the coordinate plane.

Example 5

Find the perimeter of $\triangle ABC$, given $A(-1, 1)$, $B(3, 2)$, and $C(3, -1)$. Round all measures to one decimal place.

Solution:

First plot the points and join them to form $\triangle ABC$.



Next find the length of each side.
Use the distance formula for AB and AC .

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{Let } A(-1, 1) = (x_1, y_1).$$

$$\text{Let } B(3, 2) = (x_2, y_2).$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AB = \sqrt{[3 - (-1)]^2 + (2 - 1)^2}$$

$$= \sqrt{(3+1)^2 + (1)^2}$$

$$= \sqrt{4^2 + 1}$$

$$= \sqrt{16 + 1}$$

$$= \sqrt{17}$$

$$\approx 4.123\ 105\ 626$$

$$\approx 4.1 \text{ (rounded to one decimal place)}$$

$$\text{Let } A(-1, 1) = (x_1, y_1).$$

$$\text{Let } C(3, -1) = (x_2, y_2).$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$AC = \sqrt{[3 - (-1)]^2 + (-1 - 1)^2}$$

$$= \sqrt{(3+1)^2 + (-2)^2}$$

$$= \sqrt{4^2 + 4}$$

$$= \sqrt{16 + 4}$$

$$= \sqrt{20}$$

$$\approx 4.472\ 135\ 955$$

$$\approx 4.5 \text{ (rounded to one decimal place)}$$

Line segment BC is a vertical line segment.
The length of this line segment is the difference between the y -values.

$$d = y_2 - y_1$$

$$BC = 2 - (-1)$$

$$= 2 + 1$$

$$= 3$$

You can also see that the rise of $\overline{BC}$ is 3.

$$\text{Perimeter} = AB + AC + BC$$

$$= 4.1 + 4.5 + 3$$

$$= 11.6$$

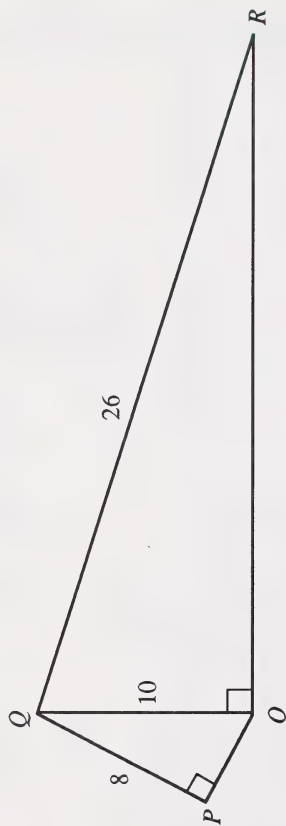
The perimeter of $\triangle ABC$ is 11.6 units.

Keep in mind that perimeter is the distance around a geometric figure. Use linear units to show perimeter. You should also remember that any distance measurement is always positive.

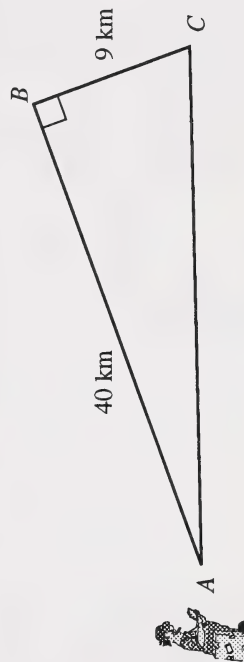


Now do any five of the following eight questions.

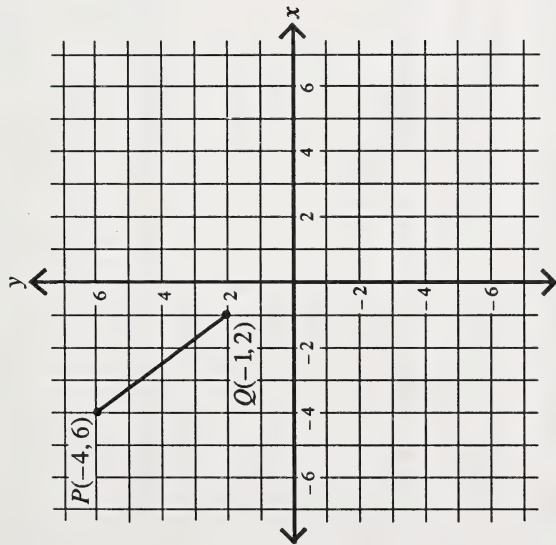
1. In this figure $\angle OPQ$ and $\angle QOR$ are right angles. Calculate the measures of $\overline{OP}$ and $\overline{OR}$.



2. A courier from Town A had to make deliveries to Town B and Town C. The courier went to Town B first and then went on to Town C. On the return journey, the courier travelled via a shortcut from Town C to Town A. This shortcut is shown in the diagram. How much distance did the courier save by returning via the shortcut?



3. Calculate the distance between $P(-4, 6)$ and $Q(-1, 2)$.

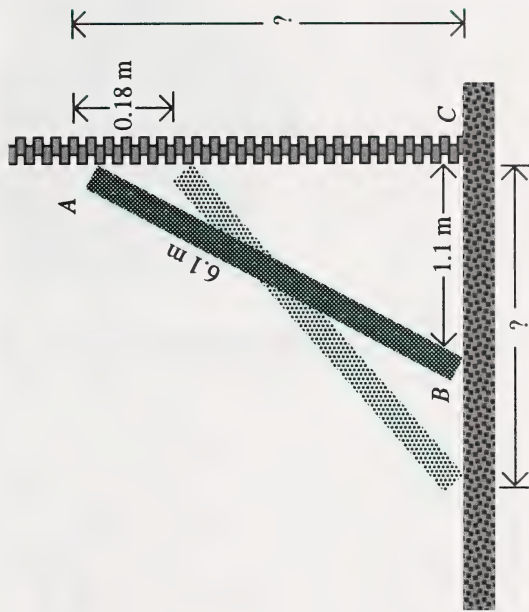


4. Use a calculator to find side PR of a right triangle if side QR is 15.2 m and hypotenuse PQ is 22.80 m. Express your answer to the nearest whole number.
5. Use the distance formula to find the distance between $A(-3, 8)$ and $B(5, 7)$. Round your answer to the nearest hundredth.
6. Find the perimeter of $\triangle PQR$ with vertices $P(-3, -8)$, $Q(5, 7)$, and $R(5, -8)$.

7. A seaport is located at $P(-9, 4)$, a fishing boat is located at $B(3, 7)$, and a yacht is located at $Y(7, -5)$. If each unit represents a sea mile or knot, calculate the following:

- the distance between the boat and the yacht rounded to two decimal places
- the distance between the port and the boat rounded to two decimal places
- the distance between the port and the yacht rounded to two decimal places

8. A 6.1 m pole leans against a wall. Its foot is 1.1 m from the base of the wall. Suppose the pole slips 0.18 m down the wall.



- a. How far up the wall does the pole reach before it slips down?
- b. After the pole slips and falls 0.18 m down the wall, how far will the foot of the pole be from the base of the wall? Round your answer to the nearest hundredth or to two decimal places.



For solutions to **Activity 1**, turn to the **Appendix, Topic 4**.

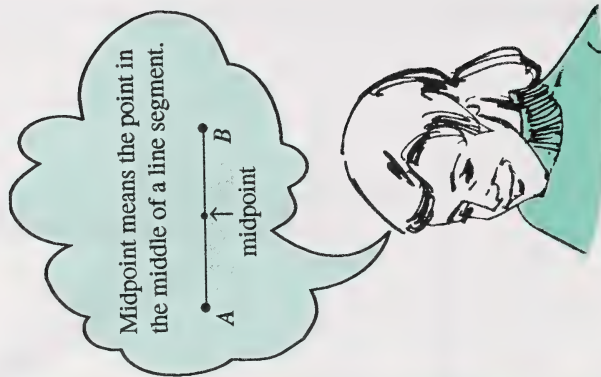
Activity 2



Use the formula $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$ to find the midpoint of a line segment.

Suppose a circle with a given diameter is drawn on a coordinate plane. The endpoints of the diameter are given and the centre of the circle needs to be found. How do you do this?

In order to find the middle of a line segment or the **midpoint**, examine some horizontal and vertical line segments.

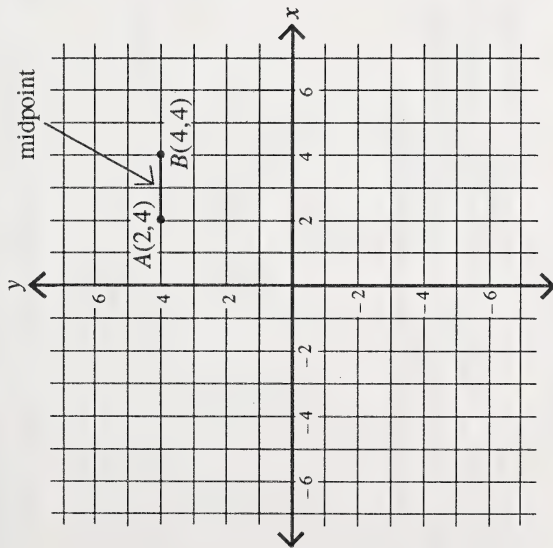


Example 6

Find the midpoint of $\overline{AB}$ with $A(2, 4)$ and $B(4, 4)$.

Solution:

Plot the points on a coordinate plane. Calculate the distance from A to B by counting.



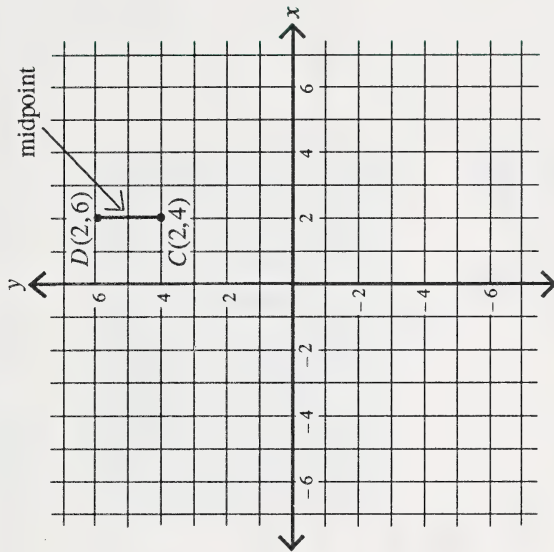
From the diagram you can see that the distance from A to B is two units. Since the midpoint is one-half of this distance, the midpoint would be at $(3, 4)$.

Example 7

Find the midpoint of $\overline{CD}$ with $C(2, 4)$ and $D(2, 6)$.

Solution:

Plot the points on a coordinate plane. Calculate the distance from C to D by counting.



From the diagram you can see that the distance from C to D is two units. Since the midpoint is one-half of this distance, the midpoint would be at $(2, 5)$.

From Examples 6 and 7 you might have noticed a method for finding the midpoint of any line segment.



To get the x -coordinate of the midpoint of a line segment, add the x -coordinates of the endpoints and divide by 2. To get the y -coordinate of the midpoint of a line segment, add the y -coordinates of the endpoints and divide by 2.

In Example 6 consider x as the coordinate of the midpoint and consider x_1 and x_2 as the first coordinates of the endpoints. Also consider y as the other coordinate of the midpoint and consider y_1 and y_2 as the second coordinates of the endpoints.

$$\begin{aligned} x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\ &= \frac{2 + 4}{2} & &= \frac{4 + 4}{2} \\ &= \frac{6}{2} & &= \frac{8}{2} \\ &= 3 & &= 4 \end{aligned}$$

The coordinates for the midpoint of line segment AB are $(3, 4)$ if $A(2, 4)$ and $B(4, 4)$ are the endpoints.

In Example 7 consider x as the first coordinate of the midpoint and x_1 and x_2 as the first coordinates of the endpoints. Also consider y as the second coordinate of the midpoint and y_1 and y_2 as the second coordinates of the endpoints.

$$\begin{aligned} x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\ &= \frac{2 + 2}{2} & &= \frac{4 + 6}{2} \\ &= \frac{4}{2} & &= \frac{10}{2} \\ &= 2 & &= 5 \end{aligned}$$

The coordinates for the midpoint of line segment CD are $(2, 5)$ if $C(2, 4)$ and $D(2, 6)$ are the endpoints.

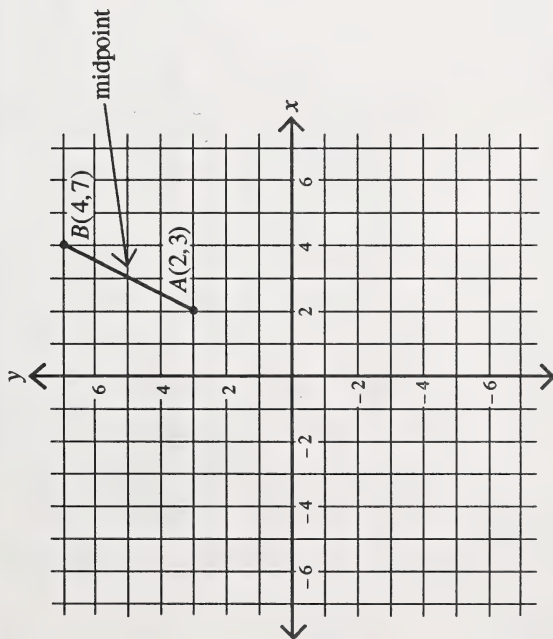
Example 8

Find the coordinates of the midpoint (x, y) of the line segment joining the points $A(2, 3)$ and $B(4, 7)$.

Solution:

$$\begin{aligned} x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\ &= \frac{2 + 4}{2} & &= \frac{3 + 7}{2} \\ &= \frac{6}{2} & &= \frac{10}{2} \\ &= 3 & &= 5 \end{aligned}$$

Plot the points $A(2, 3)$ and $B(4, 7)$ on a coordinate plane.



From the diagram it appears that $(3, 5)$ is the midpoint of $\overline{AB}$. Use the formula to see if this is correct.

The coordinates of the midpoint of $\overline{AB}$ are $(3, 5)$.

If $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are the endpoints of a line segment, then the midpoint (x, y) is found by using the following formula:

$$\left(x = \frac{x_1 + x_2}{2}, y = \frac{y_1 + y_2}{2} \right)$$

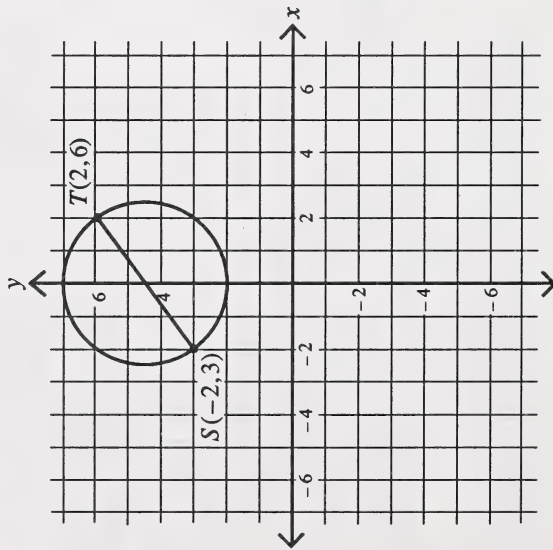


Example 9

Find the centre of a circle when the endpoints of the diameter are $S(-2, 3)$ and $T(2, 6)$.

Solution:

Plot the points on a coordinate plane. Sketch a circle so that the endpoints of the diameter are S and T .



$$\begin{aligned}x &= \frac{x_1 + x_2}{2} \\&= \frac{-2 + 2}{2} \\&= \frac{0}{2} \\&= 0\end{aligned}$$
$$\begin{aligned}y &= \frac{y_1 + y_2}{2} \\&= \frac{3 + 6}{2} \\&= \frac{9}{2} \\&= 4\frac{1}{2}\end{aligned}$$

The midpoint of $\overline{ST}$ is $(0, 4\frac{1}{2})$.

Check the diagram. Does the midpoint of the diameter ST appear to be at $(0, 4\frac{1}{2})$? Yes, it does. Therefore, the centre of the circle is $(0, 4\frac{1}{2})$.

Now do any three of the following questions.

1. Find the midpoint of $\overline{AB}$ which has the endpoints $A(-3, 4)$ and $B(0, 4)$.
2. Find the midpoint of $\overline{CD}$ which has the endpoints $C(6, 5)$ and $D(6, 2)$.
3. Find the midpoint of $\overline{EF}$ which has the endpoints $E(2, 3)$ and $F(6, 1)$.
4. Find the midpoint of $\overline{GH}$ which has the endpoints $G(-2, 4)$ and $H(-1, 0)$.
5. Find the centre of the circle which has $I(0, 4)$ and $J(3, -4)$ as endpoints of the diameter.



For solutions to **Activity 2**, turn to the **Appendix, Topic 4**.

If you require help, do the Extra Help section.

If you want more challenging explorations, do the Extensions section.

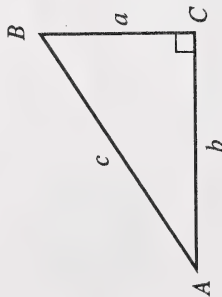
} You may decide to do both.



Extra Help

The Pythagorean Theorem

In any right triangle the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the other two sides.



The Pythagorean theorem states that $c^2 = a^2 + b^2$.

Triangle ABC is a right triangle where $\angle C = 90^\circ$. The measure of each side is denoted by the small letter corresponding to the angle **opposite** that side. Thus, side a is opposite $\angle A$, side b is opposite $\angle B$, and c is the measure of the hypotenuse, which is the side opposite the right angle, $\angle C$.

The Pythagorean theorem works especially well for certain natural numbers such as 3, 4, and 5 or 5, 12, and 13. Such numbers are called Pythagorean triples. If a triangle has sides of Pythagorean triples or a multiple of Pythagorean triples, the triangle will be a right triangle. For example, a triangle with sides of $13 \times \textcircled{3} = 39$ cm, $12 \times \textcircled{3} = 36$ cm, and $5 \times \textcircled{3} = 15$ cm is a right triangle.

The calculations show this.

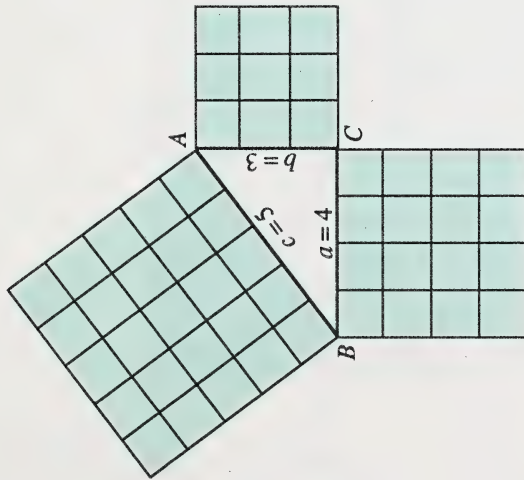
$$39^2 = 36^2 + 15^2$$

$$1521 = 1296 + 225$$

$$1521 = 1521$$

Now try the following question.

- Right triangle ABC is drawn as shown with $a = 4$ units, $b = 3$ units, and $c = 5$ units.



Fill in the blanks.

- The area of the square drawn on the hypotenuse is $c^2 = 5^2 =$ _____ square units.
- The areas of the squares drawn on the other two sides are $a^2 =$ _____ $b^2 =$ _____ square units and $b^2 =$ _____ square units.

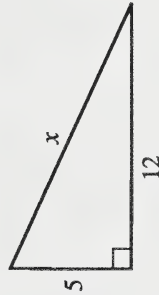
c. $c^2 = a^2 + b^2$
 $5^2 =$ _____ $+$ _____
 $25 =$ _____ $+$ _____
 $25 =$ _____



For solutions to **Extra Help**, turn to the **Appendix, Topic 4**.

Example 10

Find the value of x .



Solution:

Note that x represents the length of the hypotenuse since it is the length of the side opposite the right angle.

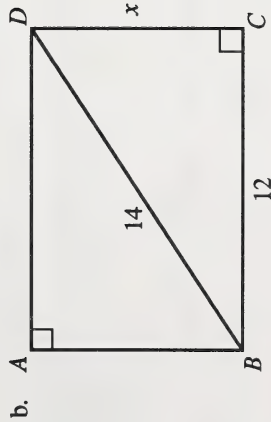
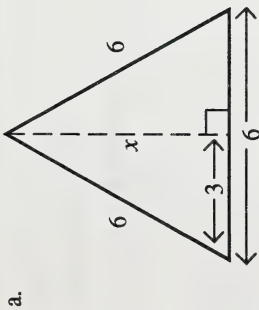
$$\begin{aligned} x^2 &= 5^2 + 12^2 \\ x^2 &= 25 + 144 \\ x^2 &= 169 \\ \sqrt{x^2} &= \sqrt{169} \\ x &= 13 \end{aligned}$$

Now try some more questions on your own.

- Complete the chart to determine which of the following are Pythagorean triples.

Triple (a, b, c)	a^2	b^2	$a^2 + b^2$	c^2	Does $c^2 = a^2 + b^2$?
(6, 8, 10)	$6^2 = 36$	$8^2 = 64$	$36 + 64 = 100$	$10^2 = 100$	Yes
(4, 5, 6)					
(12, 15, 18)					
(9, 12, 15)					
(5, 8, 9)					
(10, 24, 26)					
(1, 2, $\sqrt{3}$)				$(\sqrt{3})^2 = 3$	

3. Find the value of x . Round your answer to one decimal place.



For solutions to **Extra Help**, turn to the **Appendix, Topic 4**.



Extensions

When using the distance formula to find the distance between two points on a coordinate plane, the coordinates for x and y do not have to be whole numbers. Study the examples that follow to see how approximate distances can be calculated when the coordinates are fractional numbers, decimal numerals, or radicals. Extensive use of a calculator is recommended when working with these types of values.

Example 11

Find the distances between points A and B when the coordinates for these points are as follows. Approximate to the nearest hundredth using a calculator.

- $A = \left(2\frac{3}{4}, -4\frac{1}{2}\right)$ and $B = \left(-3\frac{1}{3}, 5\frac{2}{5}\right)$

Solution:

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{\left(-3\frac{1}{3} - 2\frac{3}{4}\right)^2 + \left[5\frac{2}{5} - \left(-4\frac{1}{2}\right)\right]^2} \\
 &= \sqrt{\left(-3\frac{4}{12} - 2\frac{9}{12}\right)^2 + \left(5\frac{2}{5} + 4\frac{1}{2}\right)^2} \\
 &= \sqrt{\left(-5\frac{13}{12}\right)^2 + \left(5\frac{4}{10} + 4\frac{5}{10}\right)^2} \\
 &= \sqrt{\left(-6\frac{1}{12}\right)^2 + \left(9\frac{9}{10}\right)^2} \\
 &= \sqrt{\left(\frac{-73}{12}\right)^2 + \left(\frac{99}{10}\right)^2} \\
 &\doteq \sqrt{(-6.08\bar{3})^2 + (9.9)^2} \\
 &\doteq \sqrt{37.006\,944\,44 + 98.01} \\
 &\doteq \sqrt{135.016\,944\,44} \\
 &\doteq 11.619\,679\,19 \\
 &\doteq 11.62
 \end{aligned}$$

The distance between points A and B is approximately 11.62 or $11\frac{31}{50}$ units.

- $A = (4.36, 6.04)$ and $B = (-2.52, -6.44)$

Solution:

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-2.52 - 4.36)^2 + (-6.44 - 6.04)^2} \\
 &= \sqrt{(-6.88)^2 + (-12.48)^2} \\
 &= \sqrt{47.3344 + 155.7504} \\
 &= \sqrt{203.0848} \\
 &\doteq 14.250\,782\,43 \\
 &\doteq 14.25
 \end{aligned}$$

The distance between points A and B is approximately 14.25 units.

It is time to try some similar problems on your own. Do two of the following questions.

In each of the following find the distance between points A and B using the distance formula.

1. $A = \left(-4\frac{1}{3}, -6\frac{1}{2}\right)$ $B = \left(5\frac{1}{2}, -1\frac{3}{4}\right)$

2. $A = \left(6\frac{1}{5}, 3\frac{1}{4}\right)$ $B = \left(-1\frac{1}{8}, -6\frac{5}{9}\right)$

3. $A = (-4.3, -6.5)$ $B = (5.5, -1.75)$

4. $A = (6.2, 3.25)$ $B = (-1.125, -6.56)$

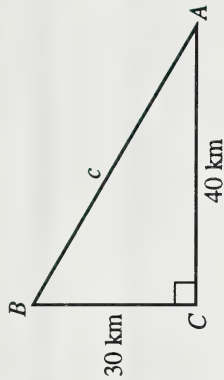


For solutions to **Extensions**, turn to the **Appendix**, **Topic 4**.

Sometimes you may be asked to determine how long it takes to go a certain distance. If you are given the rate of speed (r) and the distance travelled (d), you can calculate the time required (t) using the formula $t = \frac{d}{r}$. Study the next example and then, using your knowledge of the Pythagorean theorem, answer the question that follows.

Example 12

How long does it take to drive from A to B assuming that the rate of speed is 80 km/h?



Solution:

Use the Pythagorean theorem to find the distance from A to B .

$$c^2 = a^2 + b^2$$

$$c^2 = 30^2 + 40^2$$

$$c^2 = 900 + 1600$$

$$c^2 = 2500$$

$$c = \sqrt{2500}$$

$$= 50 \text{ km}$$

Then use the formula $t = \frac{d}{r}$.

$$t = \frac{d}{r}$$

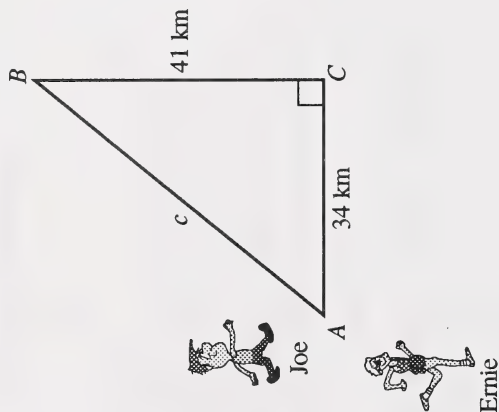
$$= \frac{50 \text{ km}}{80 \text{ km/h}}$$

$$= \frac{5}{8} \text{ h or } 0.625 \text{ h}$$

To change 0.625 h to minutes, multiply 0.625 by 60 to get $0.625 \times 60 = 37.5$ min. It would take $\frac{5}{8}$ h or 37.5 min to drive from A to B .

Try the following question.

5. Joe is going to walk directly from Point A to Point B at the rate of 5 km/h . Ernie is going to run from Point A to Point C and then to Point B at the rate of 7.5 km/h . If they leave at the same time, who will get to Point B first?



For solutions to **Extensions**, turn to the **Appendix, Topic 4**.

Unit Summary



What You Have Learned

In this unit you studied how to do the following:

- Graph and name coordinates.
- Use the slope formula to find positive, negative, zero, and undefined slopes.
- Verify relationships between the slopes of parallel and perpendicular lines and apply these to problem-solving situations.
- Plot several points and draw the graph of a linear equation.
- Write and graph the equation of a line in the form $y = mx + b$.
- Use the Pythagorean theorem and the distance formula to find the distance between two points.
- Find the midpoint of a line segment.

You are now ready to
complete the **Unit Assignment**.

Appendix



Solutions

Review

Topic 1 Graphing and Naming Coordinates

Topic 2 Slope of a Line

Topic 3 Graphing Linear Equations

Topic 4 Distance Between Two Points



Review

1. a.



$$c^2 = a^2 + b^2$$

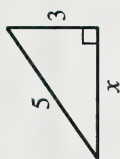
$$x^2 = 5^2 + 12^2$$

$$x^2 = 25 + 144$$

$$x^2 = 169$$

$$\sqrt{x^2} = \sqrt{169}$$

$$x = 13 \text{ units}$$



$$c^2 = a^2 + b^2$$

$$5^2 = 3^2 + x^2$$

$$25 = 9 + x^2$$

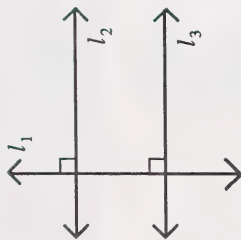
$$x^2 = 25 - 9$$

$$x^2 = 16$$

$$\sqrt{x^2} = \sqrt{16}$$

$$x = 4 \text{ units}$$

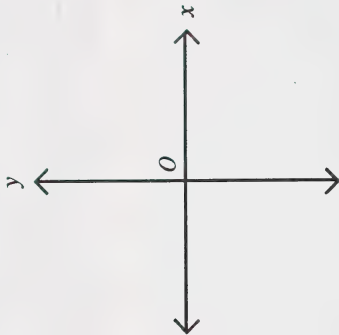
2.



In this diagram the parallel lines and the perpendicular lines are as follows:

- Lines l_2 and l_3 are parallel lines. This can also be written as $l_2 \parallel l_3$.
- Line l_1 is perpendicular to line l_2 . This can also be written as $l_1 \perp l_2$.
- Line l_1 is perpendicular to line l_3 . This can also be written as $l_1 \perp l_3$.

3.



- The vertical line is called the y-axis.
- The horizontal line is called the x-axis.
- The point O where the lines cross each other is called the origin.

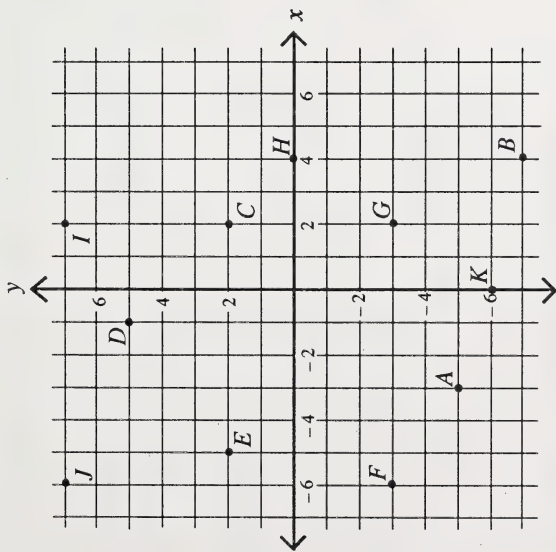


Exploring Topic 1

Activity 1

Graph and name coordinates.

1. The coordinates are $A(-3, -5)$, $B(4, -7)$, $C(2, 2)$, $D(-1, 5)$, and $E(-5, 2)$.

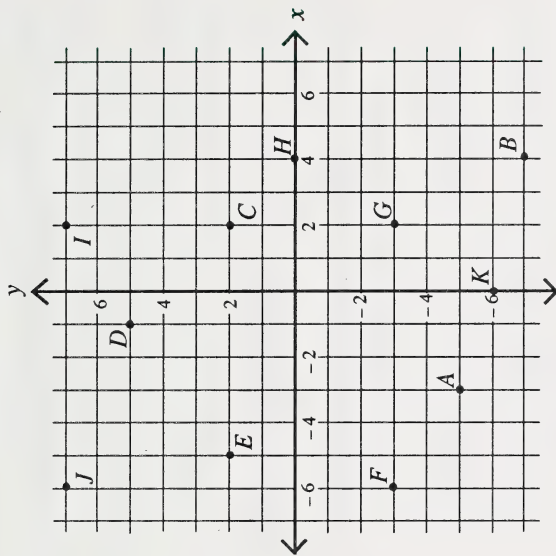


2. a. For $(-6, 7)$, the point is J .
 b. For $(-3, -5)$, the point is A .
 c. For $(2, 2)$, the point is C .
 d. For $(0, -6)$, the point is K .

- e. For $(2, 7)$, the point is I .

- f. For $(-5, 2)$, the point is E .

3.



Points $C(2, 2)$ and $I(2, 7)$ are in Quadrant I.

Points $D(-1, 5)$, $E(-5, 2)$, and $J(-6, 7)$ are in Quadrant II.

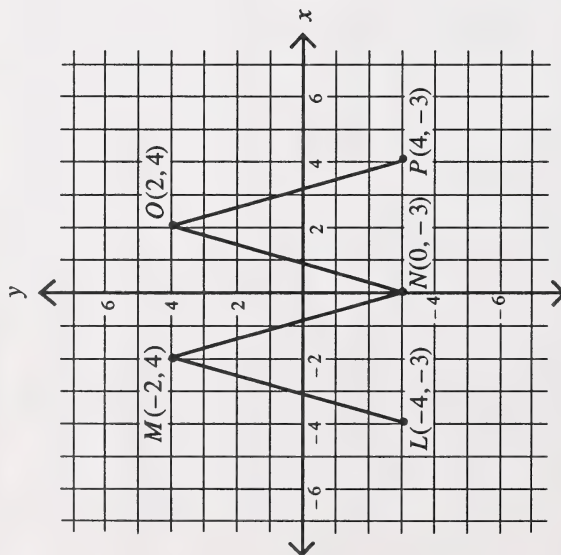
Points $A(-3, -5)$ and $F(-6, -3)$ are in Quadrant III.

Points $B(4, -7)$ and $G(2, -3)$ are in Quadrant IV.

The other points lie on an axis. Point $H(4, 0)$ is on the x -axis and point $K(0, -6)$ is on the y -axis.

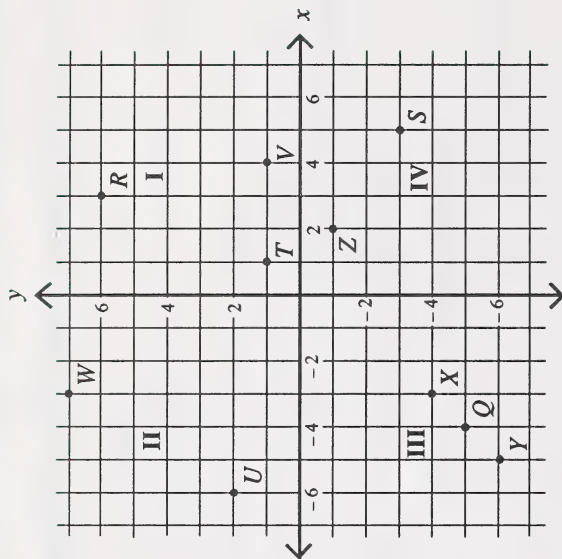
4. a. When a point lies on the x -axis, the second component is zero.
- b. When a point lies on the y -axis, the first component is zero.
- c. When a point is not on either of the axes, both components of the ordered pair are nonzero.

5.



When the points are joined according to the directions, the result is the letter M.

6.



- a. Point $Q(-4, -5)$ is in Quadrant III.
- b. Point $R(3, 6)$ is in Quadrant I.
- c. Point $S(5, -3)$ is in Quadrant IV.
- d. Point $T(1, 1)$ is in Quadrant I.
- e. Point $U(-6, 2)$ is in Quadrant II.
- f. Point $V(4, 1)$ is in Quadrant I.

- g. Point $W(-3, 7)$ is in Quadrant II.
- h. Point $X(-3, -4)$ is in Quadrant III.
- i. Point $Y(-5, -6)$ is in Quadrant III.
- j. Point $Z(2, -1)$ is in Quadrant IV.
- Extra Help**
- a. A point has no dimension. It has no length, width, or thickness.

b. A line has one dimension. It has length but no width or thickness.

c. A plane has length and width but no thickness. It has two dimensions.

d. Space has three dimensions. It has length, width, and thickness.
 - The vertical and horizontal lines that intersect at point $A(-4, 5)$ are $x = -4$ and $y = 5$.
 - a. The origin is the point of intersection of the x - and y -axes. It is the centre of the Cartesian plane. The ordered pair $(0, 0)$ represents the origin.

b. Coordinates are ordered pairs that locate points on a graph.

c. Ordered pairs position or locate points in a coordinate plane.

d. The ordinate is the y -component of the ordered pair.

e. The abscissa is the x -component of the ordered pair.
 - a. The line described by $x = 0$ is the y -axis.

b. The line described by $x = -4$ is a line parallel to the y -axis which passes through the point $(-4, 0)$.

c. The line described by $y = 0$ is the x -axis.

d. The line described by $y = 3$ is a line parallel to the x -axis which passes through the point $(0, 3)$.
 - a. Point $A(1, -4)$ is found in Quadrant IV.

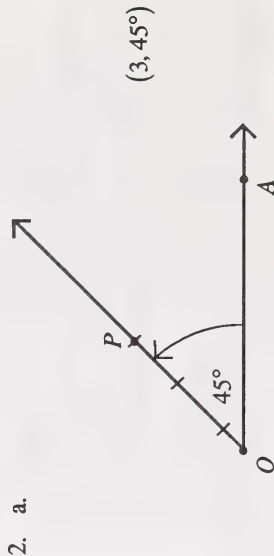
b. Point $B(-4, -4)$ is found in Quadrant III.

c. Point $C(-2, 4)$ is found in Quadrant II.

d. Point $D(2, 4)$ is found in Quadrant I.

Extensions

1. a. The pole is any arbitrary point in the plane that is used as a reference. It compares with the origin in the Cartesian plane.
- b. The polar axis is a reference line drawn from the pole. It is generally a horizontal ray with the pole as its endpoint. It is also called the initial arm.
- c. The terminal arm is any ray that defines the extent of an angle which is measured from the polar axis.

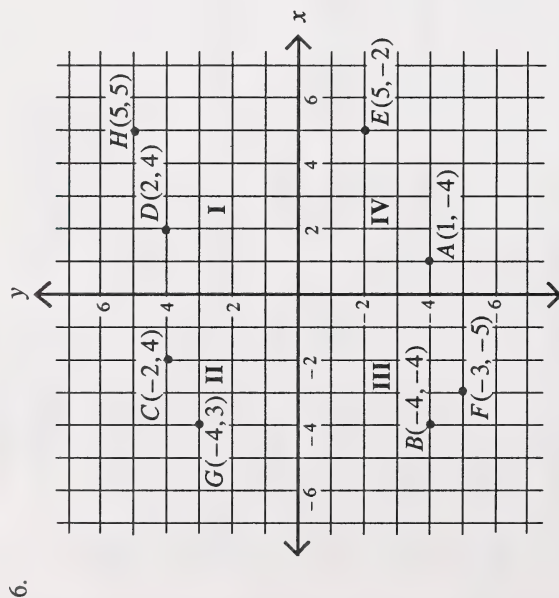


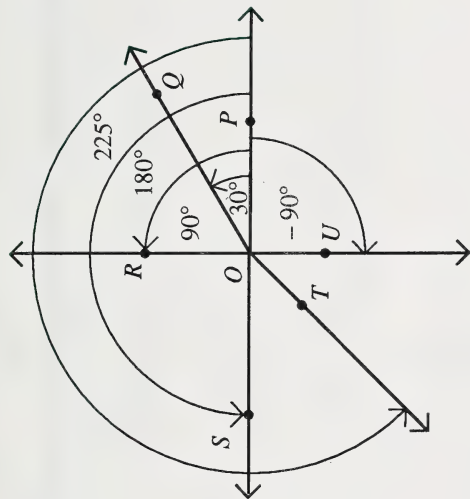
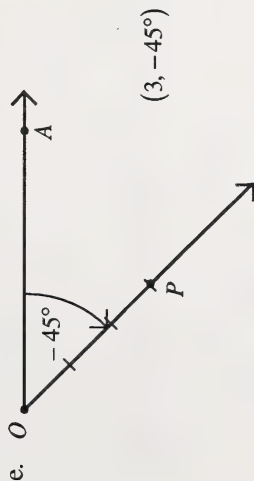
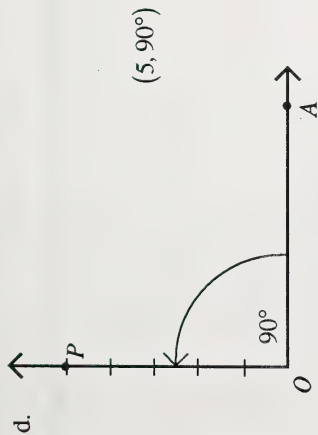
e. Point $E(5, -2)$ is found in Quadrant IV.

f. Point $F(-3, -5)$ is found in Quadrant III.

g. Point $G(-4, 3)$ is found in Quadrant II.

h. Point $H(5, 5)$ is found in Quadrant I.





Since $OP = 3.8$, it follows that the polar coordinate is

$P(3.8, 0^\circ)$.

Since $OQ = 7$, it follows that the polar coordinate is $Q(7, 30^\circ)$.

Since $OR = 4$, it follows that the polar coordinate is $R(4, 90^\circ)$.

Since $OS = 6$, it follows that the polar coordinate is $S(6, 180^\circ)$.

Since $OT = 2.5$, it follows that the polar coordinate is

$T(2.5, 225^\circ)$.

Since $OU = 3$, it follows that the polar coordinate is $U(3, -90^\circ)$

or $U(3, 270^\circ)$.

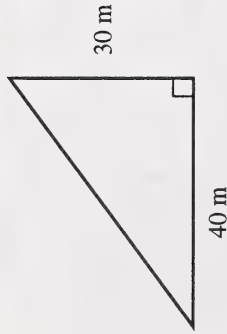


Exploring Topic 2

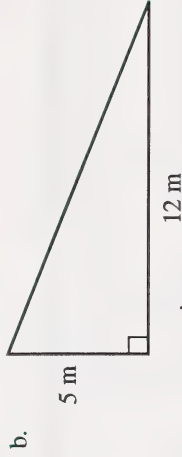
Activity 1

Use the slope formula to find positive, negative, zero, and undefined slopes.

1. a.



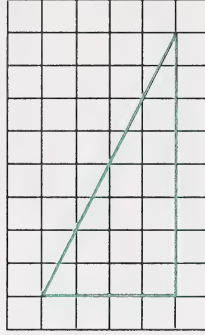
$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{30}{40} \\ &= \frac{3}{4} \\ &= 0.75\end{aligned}$$



$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{5}{12} \\ &= 0.417 \quad (\text{rounded to three decimal places})\end{aligned}$$

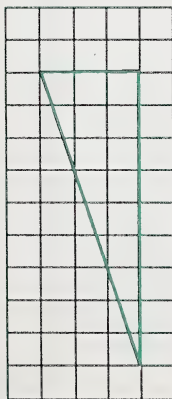
2. Since the question is on the grid, you have to be concerned with the sign of the slope. Squares are counted from left to right.

a.



$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{number of squares up}}{\text{number of squares across}} \\ &= -\frac{4}{8} \\ &= -\frac{1}{2} \quad \text{or} \quad -0.5\end{aligned}$$

The slope is negative because the oblique line falls to the right.

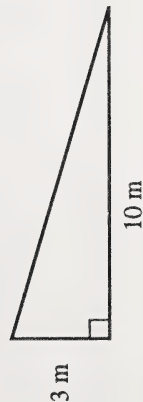


b.

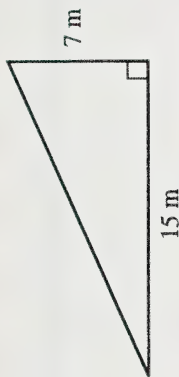
$$\begin{aligned}\text{Slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{number of squares up}}{\text{number of squares across}} \\ &= \frac{3}{3} \\ &= 1 \\ &= \frac{1}{1} \\ &= 1\end{aligned}$$

The slope is positive because the oblique line rises to the right.

3. a.



$$\begin{aligned}\text{Pitch} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{3}{10} \\ &= 0.3\end{aligned}$$



b.

$$\begin{aligned}\text{Pitch} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{7}{15} \\ &= 0.467 \quad (\text{rounded to three decimal places})\end{aligned}$$

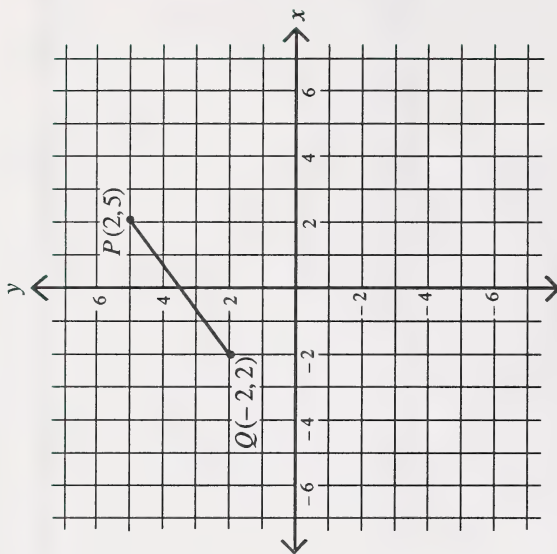
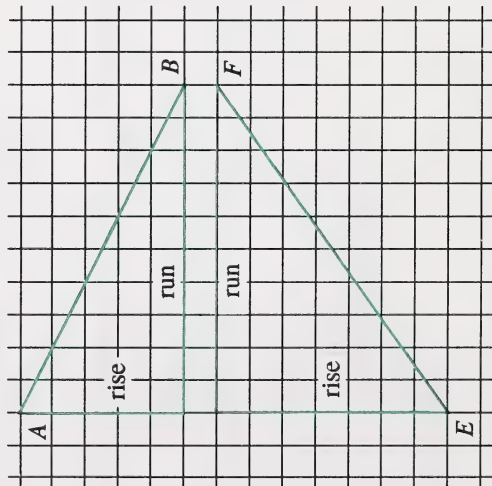
$$\begin{aligned}4. \text{ Slope of } \overline{AB} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\text{squares up}}{\text{squares across}} \\ &= -\frac{5}{10} \\ &= -0.5\end{aligned}$$

The slope is negative because the oblique line falls to the right.

$$\begin{aligned}
 \text{Slope of } \overline{EF} &= \frac{\text{rise}}{\text{run}} \\
 &= \frac{\text{squares up}}{\text{squares across}} \\
 &= \frac{7}{10} \\
 &= 0.7
 \end{aligned}$$

The slope is positive because the oblique line rises to the right.

Therefore, $\overline{AB}$ has a negative slope and $\overline{EF}$ has a positive slope.



5.

Let $Q(-2, 2)$ be (x_1, y_1) .

Let $P(2, 5)$ be (x_2, y_2) .

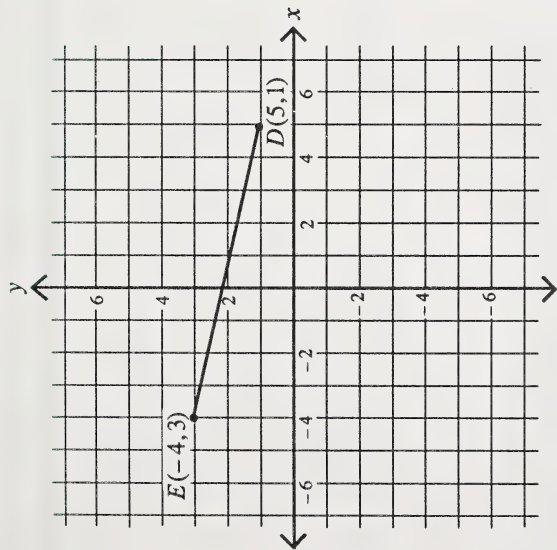
$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{5 - 2}{2 - (-2)}$$

$$= \frac{3}{4}$$

$$= 0.75$$

6. a.



Let $D(5, 1)$ be (x_1, y_1) .

Let $E(-4, 3)$ be (x_2, y_2) .

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

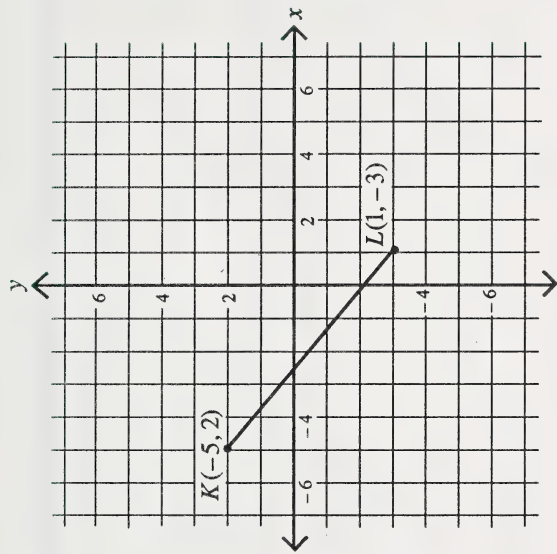
$$\text{Slope of } \overline{DE} = \frac{3 - 1}{-4 - 5}$$

$$= \frac{2}{-4 + (-5)}$$

$$= \frac{2}{-9}$$

$$= -0.222 \quad (\text{rounded to three decimal places})$$

b.



Let $L(1, -3)$ be (x_1, y_1) .

Let $K(-5, 2)$ be (x_2, y_2) .

$$\begin{array}{ccc} K(-5, 2) & L(1, -3) \\ \updownarrow & \updownarrow \\ (x_2, y_2) & (x_1, y_1) \end{array}$$

$$\text{Slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

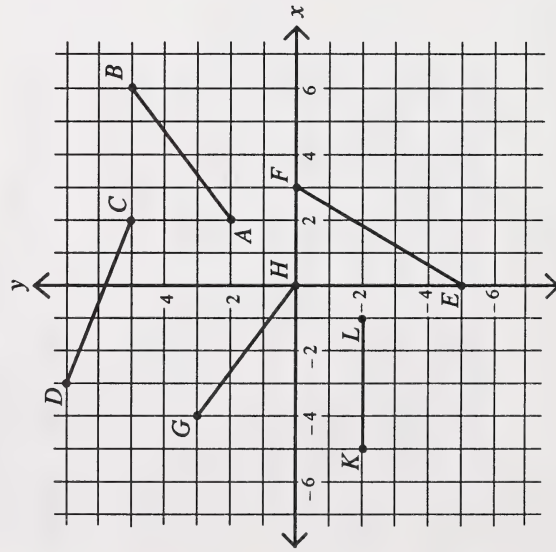
$$\text{Slope of } \overline{KL} = \frac{2 - (-3)}{-5 - 1}$$

$$= \frac{2+3}{-6}$$

$$= \frac{-5}{6}$$

$$= -0.833 \quad (\text{rounded to three decimal places})$$

7. Use the method of counting squares. Squares are counted from left to right on the coordinate plane.



$$\begin{aligned} \text{Slope of } \overline{AB} &= \frac{3}{4} \\ &= 0.75 \end{aligned}$$

$$\begin{aligned} \text{Slope of } \overline{CD} &= -\frac{2}{5} \\ &= -0.4 \end{aligned}$$

$$\begin{aligned} \text{Slope of } \overline{EF} &= \frac{5}{3} \\ &= 1.667 \quad (\text{rounded to three decimal places}) \end{aligned}$$

$$\begin{aligned} \text{Slope of } \overline{GH} &= -\frac{3}{4} \\ &= -0.75 \end{aligned}$$

$$\begin{aligned} \text{Slope of } \overline{KL} &= \frac{0}{4} \\ &= 0 \end{aligned}$$

8. a. $m = \frac{\text{rise}}{\text{run}}$

If $m = 0$ and run = 8 cm, then $0 = \frac{\text{rise}}{8}$.

$$\therefore \text{rise} = 8 \times 0$$

$$= 0$$

The rise is zero.

b. Since the rise is zero, the line is horizontal.

9. run of ramp = 2.5 cm
slope of ramp = 1.0

$$m = \frac{\text{rise}}{\text{run}}$$

$$1 = \frac{\text{rise}}{2.5}$$

$$\text{rise} = 2.5 \times 1$$

$$\text{rise} = 2.5$$

The rise is 2.5 cm.

Activity 2

Verify the relationships between the slopes of parallel and perpendicular lines and apply these to problem-solving situations.

1.

m_1	m_2	$m_1 \times m_2$
2	$-\frac{1}{2}$	$(2)\left(-\frac{1}{2}\right) = -1$
$\frac{1}{3}$	-3	$\left(\frac{1}{3}\right)(-3) = -1$
$\frac{2}{5}$	$-\frac{5}{2}$	$\left(\frac{2}{5}\right)\left(-\frac{5}{2}\right) = -1$
undefined	0	not possible

2. Given $A(-1, 3)$ and $B(4, -4)$, the slope of $\overline{AB}$ is as follows:

$$\text{Slope of } \overline{AB} = m_1 = \frac{-4 - 3}{4 - (-1)}$$

$$= \frac{-7}{5}$$

Given $C(5, 3)$ and $D(-2, -2)$, the slope of $\overline{CD}$ is as follows:

$$\text{Slope of } \overline{CD} = m_2 = \frac{-2 - 3}{-2 - 5}$$

$$= \frac{-5}{-7}$$

$$= \frac{5}{7}$$

Since $m_1 \times m_2 = \left(\frac{-7}{5}\right)\left(\frac{5}{7}\right) = -1$, then $\overline{AB} \perp \overline{CD}$.

3. For $\triangle ABC$ to be a right triangle, either $\overline{AB} \perp \overline{BC}$, $\overline{BC} \perp \overline{AC}$, or $\overline{AB} \perp \overline{AC}$.

Let m_1 be the slope of $\overline{AB}$.

Given $A(0, -1)$ and $B(3, 2)$, the slope of $\overline{AB}$ is as follows:

$$\begin{aligned}
 m_1 &= \frac{2 - (-1)}{3 - 0} \\
 &= \frac{2 + 1}{3} \\
 &= \frac{3}{3} \\
 &= 1
 \end{aligned}$$

Let m_2 be the slope of $\overline{BC}$.

Given $B(3, 2)$ and $C(4, 1)$, the slope of $\overline{BC}$ is as follows:

$$\begin{aligned}
 m_2 &= \frac{1 - 2}{4 - 3} \\
 &= \frac{-1}{1} \\
 &= -1
 \end{aligned}$$

Since $m_1 \times m_2 = (1)(-1) = -1$, then $\triangle ABC$ is a right triangle.

You do not need to find m_3 , the slope of $\overline{AC}$, since a triangle can only have one right angle.

4. The statement $m_1 m_2 = -1$ is true only if the lines l_1 and l_2 are oblique. If the lines are vertical and horizontal, the relationship $m_1 m_2 = -1$ does not apply. Remember that vertical lines have an undefined slope and horizontal lines have a slope of zero. You cannot multiply zero by something undefined.

5. a. Given $P(4, 5)$ and $Q(2, 10)$, the slope of $\overline{PQ}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{PQ} &= \frac{10 - 5}{2 - 4} \\
 &= \frac{5}{-2} \\
 &= -2\frac{1}{2}
 \end{aligned}$$

- b. Given $M(-7, 3)$ and $N(2, -1)$, the slope of $\overline{MN}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{MN} &= \frac{-1 - 3}{2 - (-7)} \\
 &= \frac{-4}{9}
 \end{aligned}$$

- c. Given $A(0, 6)$ and $B(0, 4)$, the slope of $\overline{AB}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{AB} &= \frac{4 - 6}{0 - 0} \\
 &= \frac{-2}{0} \\
 &= \text{undefined}
 \end{aligned}$$

- d. Given $C(-2, -4)$ and $D(-8, -1)$, the slope of $\overline{CD}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{CD} &= \frac{-1 - (-4)}{-8 - (-2)} \\ &= \frac{-1 + 4}{-8 + 2} \\ &= \frac{3}{-6} \\ &= -\frac{1}{2}\end{aligned}$$

e. Given $F(9, 0)$ and $G(-5, 0)$, the slope of $\overline{FG}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{FG} &= \frac{0 - 0}{-5 - 9} \\ &= \frac{0}{-14} \\ &= 0\end{aligned}$$

6. a. If $m_1 m_2 = -1$ and $m_1 = \frac{3}{4}$, then $(\frac{3}{4})m_2 = -1$.

$$\begin{aligned}\left(\frac{3}{4}\right)m_2 &= -1 \quad \text{or} \quad m_2 = -1 \div \frac{3}{4} \\ \frac{3m_2}{4} &= -1 \quad \quad \quad = -1 \times \frac{4}{3} \\ 3m_2 &= -4 \quad \quad \quad = \frac{-4}{3} \\ m_2 &= -\frac{4}{3}\end{aligned}$$

Note that $\frac{3}{4}$ and $-\frac{4}{3}$ are negative reciprocals.

b. If $m_1 m_2 = -1$ and $m_1 = 0$, then $m_2(0) = -1$.

$$\begin{aligned}m_2(0) &= -1 \quad \text{or} \quad m_2 = -1 \div 0 \\ m_2 &= -\frac{1}{0}\end{aligned}$$

Thus, m_2 is undefined.

c. If $m_1 m_2 = -1$ and $m_1 = -\frac{3}{5}$, then $m_2(-\frac{3}{5}) = -1$.

$$\begin{aligned}m_2\left(-\frac{3}{5}\right) &= -1 \quad \text{or} \quad m_2 = -1 \div \frac{-3}{5} \\ \frac{-3m_2}{5} &= -1 \quad \quad \quad = -1 \times \frac{5}{-3} \\ -3m_2 &= -5 \quad \quad \quad = \frac{5}{3} \\ m_2 &= \frac{-5}{-3} \\ &= \frac{5}{3}\end{aligned}$$

Note that $-\frac{3}{5}$ and $\frac{5}{3}$ are negative reciprocals.

d. If $m_1 m_2 = -1$ and $m_1 = 3$, then $m_2(3) = -1$.

$$\begin{aligned}m_2(3) &= -1 \quad \text{or} \quad m_2 = -1 \div 3 \\ \frac{m_2(3)}{3} &= -\frac{1}{3} \quad \quad \quad = -1 \times \frac{1}{3} \\ m_2 &= -\frac{1}{3} \quad \quad \quad = \frac{-1}{3}\end{aligned}$$

Note that 3 and $-\frac{1}{3}$ are negative reciprocals.

e. If $m_1 m_2 = -1$ and $m_1 = 1$, then $m_2(1) = -1$.

$$m_2(1) = -1 \quad \text{or} \quad m_2 = -1 + 1$$

$$m_2 = \frac{-1}{1} = -1$$

$$= -1$$

Note that -1 and 1 are negative reciprocals.

7. a. Given $C(7, 2)$ and $D(5, 9)$, the slope of $\overline{CD}$ is as follows:

$$\text{Slope of } \overline{CD} = \frac{9-2}{5-7}$$

$$= \frac{7}{-2}$$

$$= \frac{-7}{2}$$

b. Given $X(2, -1)$ and $Y(-7, 3)$, the slope of $\overline{XY}$ is as follows:

$$\text{Slope of } \overline{XY} = \frac{3-(-1)}{-7-2}$$

$$= \frac{4}{-9}$$

$$= \frac{-4}{9}$$

c. Given $A(4, 9)$ and $B(1, 3)$, the slope of $\overline{AB}$ is as follows:

$$\text{Slope of } \overline{AB} = \frac{3-9}{1-4}$$

$$= \frac{-6}{-3}$$

$$= 2$$

d. Given $M(4, -6)$ and $N(-3, -2)$, the slope of $\overline{MN}$ is as follows:

$$\text{Slope of } \overline{MN} = \frac{-2-(-6)}{-3-4}$$

$$= \frac{-2+6}{-7}$$

$$= \frac{4}{-7}$$

$$= \frac{-4}{7}$$

e. Given $R(2, 8)$ and $S(4, 1)$, the slope of $\overline{RS}$ is as follows:

$$\text{Slope of } \overline{RS} = \frac{1-8}{4-2}$$

$$= \frac{-7}{2}$$

f. Given $P(4, 5)$ and $Q(2, 1)$, the slope of $\overline{PQ}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{PQ} &= \frac{1-5}{2-4} \\
 &= \frac{-4}{-2} \\
 &= 2
 \end{aligned}$$

Therefore, $\overline{CD} \parallel \overline{RS}$ because they both have a slope of $\frac{-7}{2}$,
and $\overline{AB} \parallel \overline{PQ}$ because they both have a slope of 2.

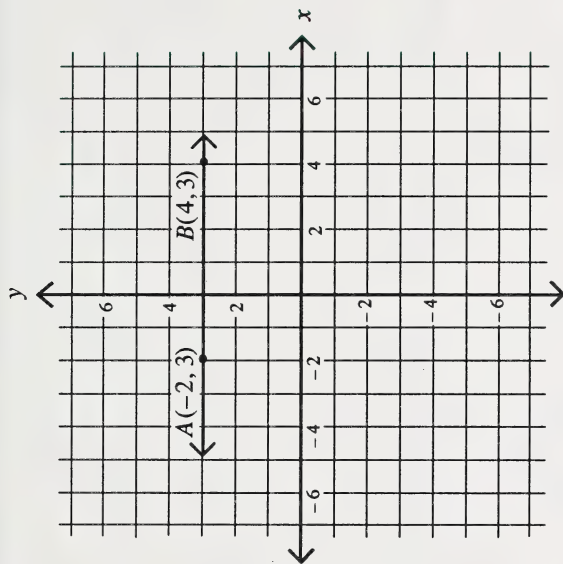
8. Given $P(6, 6)$ and $Q(2, 5)$, the slope of $\overline{PQ}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{PQ} = m_1 &= \frac{5-6}{2-6} \\
 &= \frac{-1}{-4} \\
 &= \frac{1}{4}
 \end{aligned}$$

Given $A(3, -5)$ and $B(7, -4)$, the slope of $\overline{AB}$ is as follows:

$$\begin{aligned}
 \text{Slope of } \overline{AB} = m_2 &= \frac{-4-(-5)}{7-3} \\
 &= \frac{-4+5}{7-3} \\
 &= \frac{1}{4}
 \end{aligned}$$

Since $m_1 = m_2$, $\overline{PQ}$ is parallel to $\overline{AB}$.



1. a.

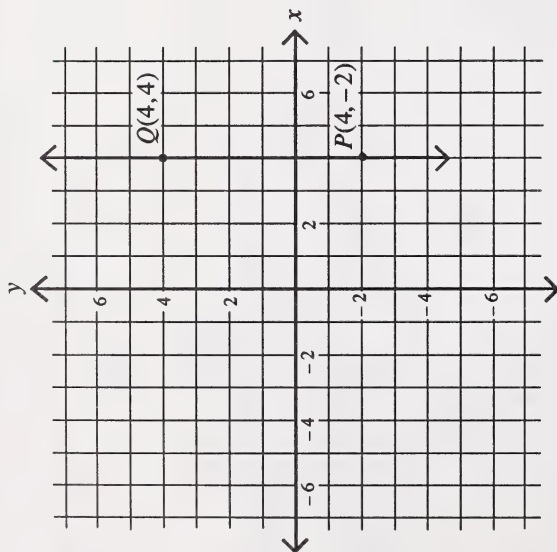
- b. The line AB is parallel to the x -axis.

$$\begin{aligned}
 \text{c. Slope of } \overleftrightarrow{AB} &= \frac{3-3}{4-(-2)} \\
 &= \frac{0}{4+2} \\
 &= \frac{0}{6} \\
 &= 0
 \end{aligned}$$

The slope of $\overleftrightarrow{AB}$ is zero.

- d. Any line that is parallel to the x -axis is horizontal and has a slope of zero.

2. a.



$$\begin{aligned} \text{b. Slope of } \overleftrightarrow{PQ} &= \frac{4 - (-2)}{4 - 4} \\ &= \frac{4 + 2}{0} \\ &= \frac{6}{0} \\ &= \text{undefined} \end{aligned}$$

The slope of $\overleftrightarrow{PQ}$ is undefined.

- c. Lines that are parallel to the y -axis have undefined slopes because you cannot divide by zero.

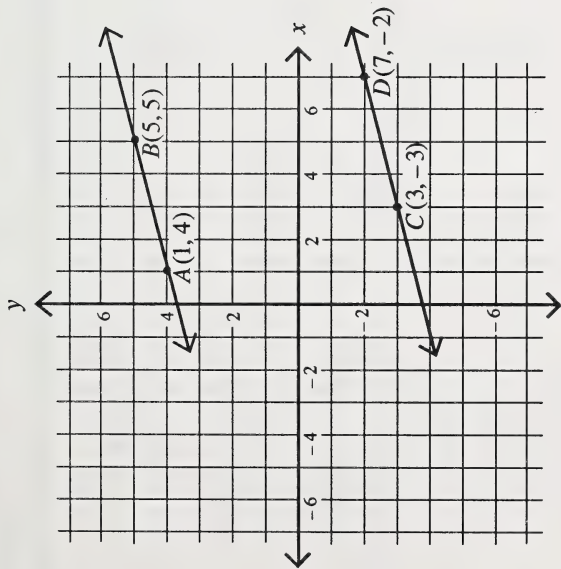
3. Given $A(1, 4)$ and $B(5, 5)$, the slope of $\overleftrightarrow{AB}$ is as follows:

$$\begin{aligned} \text{Slope of } \overleftrightarrow{AB} &= \frac{5 - 4}{5 - 1} \\ &= \frac{1}{4} \end{aligned}$$

Given $C(3, -3)$ and $D(7, -2)$, the slope of $\overleftrightarrow{CD}$ is as follows:

$$\begin{aligned} \text{Slope of } \overleftrightarrow{CD} &= \frac{-2 - (-3)}{7 - 3} \\ &= \frac{1}{4} \end{aligned}$$

Since $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ have the same slope of $\frac{1}{4}$, they are parallel.



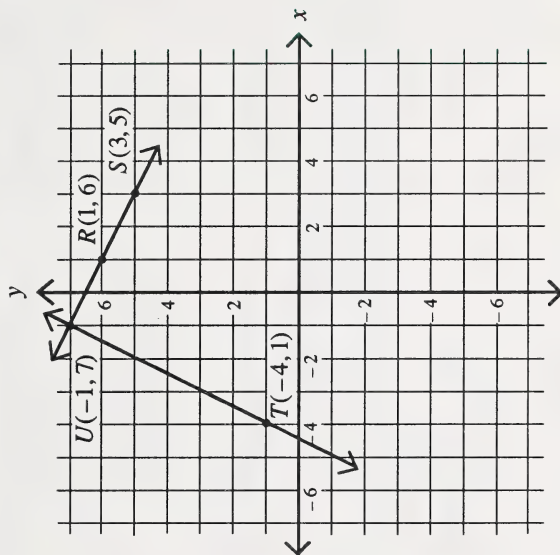
4. Given $R(1, 6)$ and $S(3, 5)$, the slope of $\overleftrightarrow{RS}$ is as follows:

$$\begin{aligned}\text{Slope of } \overleftrightarrow{RS} &= \frac{5-6}{3-1} \\ &= -\frac{1}{2}\end{aligned}$$

Given $T(-4, 1)$ and $U(-1, 7)$, the slope of $\overleftrightarrow{TU}$ is as follows:

$$\begin{aligned}\text{Slope of } \overleftrightarrow{TU} &= \frac{7-1}{-1-(-4)} \\ &= \frac{6}{3} \\ &= 2\end{aligned}$$

Since $m_1 \times m_2 = -1$ indicates that two lines are perpendicular and $\left(-\frac{1}{2}\right) \times (2) = -1$, then the line RS is perpendicular to the line TU .



Extensions

1. It is given that the slope of $\overleftrightarrow{AB} = \frac{1}{3}$ and that A is $(-2, 3b)$ and B is $(b, 4)$.

The slope of $\overleftrightarrow{AB}$ is as follows:

$$\begin{aligned}\frac{4 - 3b}{b - (-2)} &= \frac{1}{3} \\ 3(4 - 3b) &= 1[b - (-2)] \\ 12 - 9b &= b + 2 \\ 12 - 9b + 9b &= b + 9b + 2 \\ 12 &= 10b + 2 \\ 12 - 2 &= 10b + 2 - 2 \\ 10b &= 10 \\ b &= 1\end{aligned}$$

The coordinates of A are $(-2, 3b)$. Substitute 1 for b . The coordinates of A are $(-2, 3 \times 1)$ or $(-2, 3)$.

The coordinates of B are $(b, 4)$. Substitute 1 for b . The coordinates of B are $(1, 4)$.

2. Given $B(5, 18)$, $C(1, 3)$, and $D(d, 6)$, the slope of $\overline{BC}$ and the slope of $\overline{DC}$ are as follows:

$$\begin{aligned}\text{Slope of } \overline{BC} &= \frac{3 - 18}{1 - 5} \\ &= \frac{-15}{-4} \\ &= \frac{15}{4}\end{aligned}$$

$$\begin{aligned}\text{Slope of } \overline{DC} &= \frac{6 - 3}{d - 1} \\ &= \frac{3}{d - 1}\end{aligned}$$

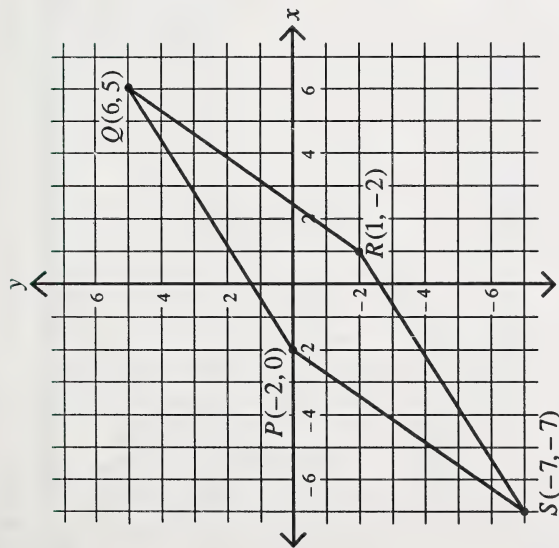
You are also told that D is on the line BC .

The slope of $\overline{BC}$ equals the slope of $\overline{DC}$ because D is on the line BC .

$$\begin{aligned}\frac{15}{4} &= \frac{3}{d - 1} \\ 15(d - 1) &= 4(3) \\ 15d - 15 &= 12 \\ 15d - 15 + 15 &= 12 + 15 \\ 15d &= 27 \\ d &= \frac{27}{15} \\ &= 1\frac{4}{5}\end{aligned}$$

The value of d is $1\frac{4}{5}$.

3. a.



Given $Q(6, 5)$ and $R(1, -2)$, the slope of $\overline{QR}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{QR} &= \frac{-2-5}{1-6} \\ &= \frac{-7}{-5} \\ &= \frac{7}{5}\end{aligned}$$

Given $R(1, -2)$ and $S(-7, -7)$, the slope of $\overline{RS}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{RS} &= \frac{-7-(-2)}{-7-1} \\ &= \frac{-5}{-8} \\ &= \frac{5}{8}\end{aligned}$$

b. Given $P(-2, 0)$ and $Q(6, 5)$, the slope of $\overline{PQ}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{PQ} &= \frac{5-0}{6-(-2)} \\ &= \frac{5}{8}\end{aligned}$$

Given $S(-7, -7)$ and $P(-2, 0)$, the slope of $\overline{PS}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{PS} &= \frac{-7-0}{-7-(-2)} \\ &= \frac{-7}{-5} \\ &= \frac{7}{5}\end{aligned}$$

- c. slope of $\overline{PQ}$ = slope of $\overline{RS}$
 slope of $\overline{QR}$ = slope of $\overline{PS}$

The slopes of the opposite sides are equal. Therefore, the opposite sides are parallel.

- d. The figure is a parallelogram or a rhombus.

4. a. Given $P(3, -4)$ and $Q(9, 2)$, the slope of $\overline{PQ}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{PQ} &= \frac{2 - (-4)}{9 - 3} \\ &= \frac{2 + 4}{6} \\ &= \frac{6}{6} \\ &= 1\end{aligned}$$

Given $R(-5, 1)$ and $S(5, 3)$, the slope of $\overline{RS}$ is as follows:

$$\begin{aligned}\text{Slope of } \overline{RS} &= \frac{3 - 1}{5 - (-5)} \\ &= \frac{2}{10} \\ &= \frac{1}{5}\end{aligned}$$

- b. In order for $\overline{PQ}$ and $\overline{RS}$ to be parallel, the slopes would have to be the same. The slopes are not equal so $\overline{PQ}$ and $\overline{RS}$ are not parallel.

In order for $\overline{PQ}$ and $\overline{RS}$ to be perpendicular, the product of their slopes would have to be -1 . Since $1 \times \frac{1}{5}$ equals $\frac{1}{5}$ and not -1 , the segments are not perpendicular.



Exploring Topic 3

Activity 1

Plot several points and draw the graph of a linear equation.

1. $2x - y - 6 = 0$

$$2x - y + y - 6 = 0 + y$$

$$2x - 6 = y$$

$$y = 2x - 6$$

When $x = -2$,

$$y = 2x - 6$$

$$= 2(-2) - 6$$

$$= -4 - 6$$

$$= -10$$

When $x = -1$,

$$y = 2x - 6$$

$$= 2(-1) - 6$$

$$= -2 - 6$$

$$= -8$$

When $x = 0$,

$$y = 2x - 6$$

$$= 2(0) - 6$$

$$= 0 - 6$$

$$= -6$$

When $x = 1$,
 $y = 2x - 6$
 $= 2(1) - 6$
 $= 2 - 6$
 $= -4$

When $x = 2$,
 $y = 2x - 6$
 $= 2(2) - 6$
 $= 4 - 6$
 $= -2$

x	-2	-1	0	1	2	3
y	-10	-8	-6	-4	-2	0

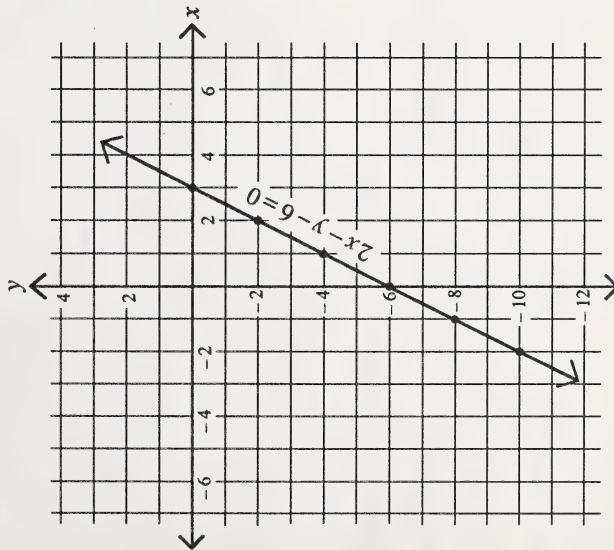
When $x = 3$,
 $y = 2x - 6$
 $= 2(3) - 6$
 $= 6 - 6$
 $= 0$

2. The x -intercept is -4 .
 The y -intercept is 3 .

The slope of the line is as follows:

$$m = \frac{\text{rise}}{\text{run}} = \frac{3}{4}$$

Note that the slope is positive because the line of the graph rises to the right.



3. $x - 3y = 0$

$$-3y = -x$$

$$y = \frac{-x}{-3}$$

$$y = \frac{x}{3}$$

When $x = -3$,

$$y = \frac{-3}{3}$$

$$= -1$$

$$= -1$$

When $x = 0$,

$$y = \frac{0}{3}$$

$$= 0$$

$$= 0$$

When $x = 3$,

$$y = \frac{3}{3}$$

$$= 1$$

$$= 1$$

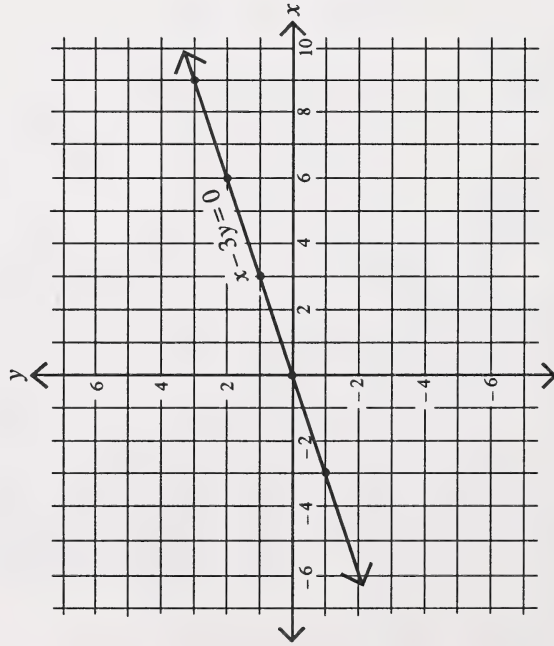
When $x = 6$,

$$y = \frac{x}{3} = \frac{6}{3} = 2$$

When $x = 9$,

$$y = \frac{x}{3} = \frac{9}{3} = 3$$

x	-3	0	3	6	9
y	-1	0	1	2	3



4. a. For the x -intercept, let $y = 0$.

When $y = 0$,

$$3x - 4(0) = 12 \\ 3x = 12 \\ x = 4$$

Thus, the x -intercept is 4.

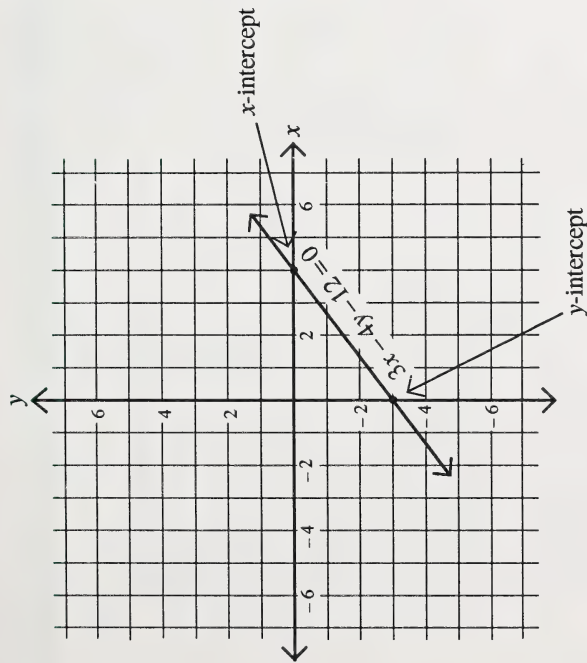
- For the y -intercept, let $x = 0$.

When $x = 0$,

$$3(0) - 4y = 12 \\ -4y = 12 \\ y = -3$$

Thus, the y -intercept is -3 .

- b. The line $3x - 4y - 12 = 0$ intersects the x -axis at the point $(4, 0)$.
- c. The line $3x - 4y - 12 = 0$ intersects the y -axis at the point $(0, -3)$.
- d. The line $3x - 4y - 12 = 0$ intersects the x -axis at $(4, 0)$, and it intersects the y -axis at $(0, -3)$. Thus, the graph of the equation is as follows.



These are found like this.

$$\begin{array}{lcl}
 2x + 3y - 6 = 0 & & 2x + 3y - 6 = 0 \\
 2(0) + 3y - 6 = 0 & & 2x + 3(0) - 6 = 0 \\
 0 + 3y - 6 = 0 & & 2x + 0 - 6 = 0 \\
 3y - 6 = 0 & & 2x - 6 = 0 \\
 3y - 6 + 6 = 0 + 6 & & 2x - 6 + 6 = 0 + 6 \\
 3y = 6 & & 2x = 6 \\
 \frac{3y}{3} = \frac{6}{3} & & \frac{2x}{2} = \frac{6}{2} \\
 y = 2 & & x = 3
 \end{array}$$

For $4x + 6y = -20$, the x - and y -intercepts are as follows:

$$\begin{array}{l}
 \text{If } x = 0, y = -3\frac{1}{3}. \\
 \text{If } y = 0, x = -5.
 \end{array}$$

5. a. For $2x + 3y - 6 = 0$, the x - and y -intercepts are as follows:

$$\begin{array}{l}
 \text{If } x = 0, y = 2. \\
 \text{If } y = 0, x = 3.
 \end{array}$$

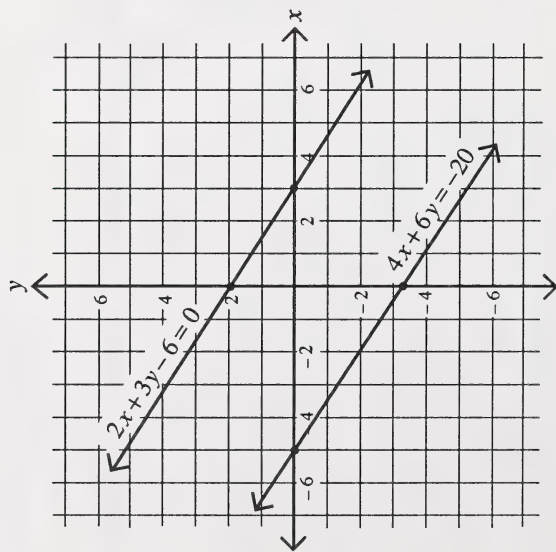
The line intersects the y -axis at $(0, 2)$, and it intersects the x -axis at $(3, 0)$.

The line intersects the y -axis at $(0, -3\frac{1}{3})$, and it intersects the x -axis at $(-5, 0)$.

These are found like this.

$$\begin{aligned}
 4x + 6y &= -20 \\
 4(0) + 6y &= -20 \\
 0 + 6y &= -20 \\
 6y &= -20 \\
 \frac{6y}{6} &= \frac{-20}{6} \\
 y &= -3\frac{1}{3}
 \end{aligned}$$

$$\begin{aligned}
 4x + 6y &= -20 \\
 4x + 6(0) &= -20 \\
 4x + 0 &= -20 \\
 4x &= -20 \\
 \frac{4x}{4} &= \frac{-20}{4} \\
 x &= -5
 \end{aligned}$$



b. The two lines are parallel. They do not meet.

c. For $2x + 3y - 6 = 0$, the x -intercept is 3 and the y -intercept is 2. For $4x + 6y = -20$, the x -intercept is -5 and the y -intercept is $-3\frac{1}{3}$.

6. $3x + 2y - 12 = 0$
 $3x + 2y - 12 + 12 = 0 + 12$

$$3x + 2y = 12$$

$$3x - 3x + 2y = 12 - 3x$$

$$2y = 12 - 3x$$

$$2y = -3x + 12$$

$$\frac{2y}{2} = \frac{-3x + 12}{2}$$

$$y = \frac{-3}{2}x + 6$$

x	-4	-2	0	2	4
y	12	9	6	3	0

When $x = -4$,

$$y = \frac{-3}{2}x + 6$$

$$= \frac{-3}{2}(-4) + 6$$

$$= -3(-2) + 6$$

$$= 6 + 6$$

$$= 12$$

When $x = -2$,

$$y = \frac{-3}{2}x + 6$$

$$= \frac{-3}{2}(-2) + 6$$

$$= -3(-1) + 6$$

$$= 3 + 6$$

$$= 9$$

When $x = 0$,

$$y = \frac{-3}{2}x + 6$$

$$= \frac{-3}{2}(0) + 6$$

$$= -3(0) + 6$$

$$= 0 + 6$$

$$= 6$$

When $x = 2$,

$$y = -\frac{3}{2}x + 6$$

$$= -\frac{3}{2}(2) + 6$$

$$= -3(1) + 6$$

$$= -3 + 6$$

$$= 3$$

When $x = 4$,

$$y = -\frac{3}{2}x + 6$$

$$= -\frac{3}{2}(4) + 6$$

$$= -3(2) + 6$$

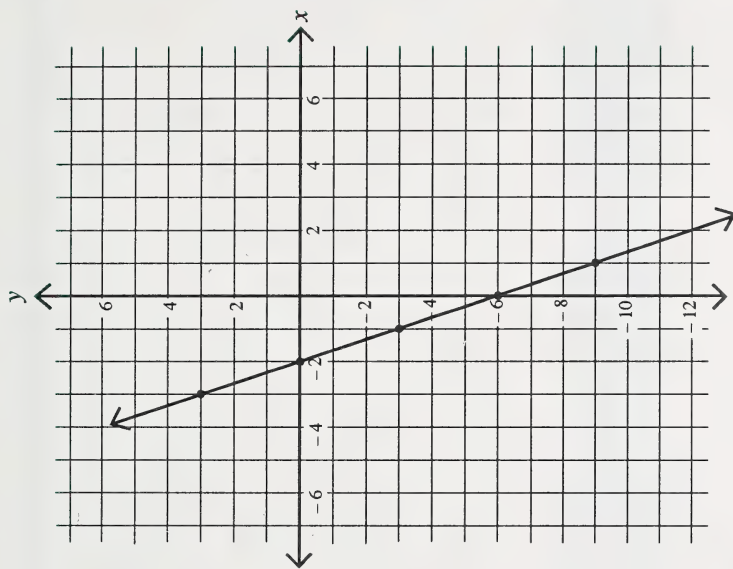
$$= -6 + 6$$

$$= 0$$

7. a.

x	-3	-2	-1	0	1
y	3	0	-3	-6	-9

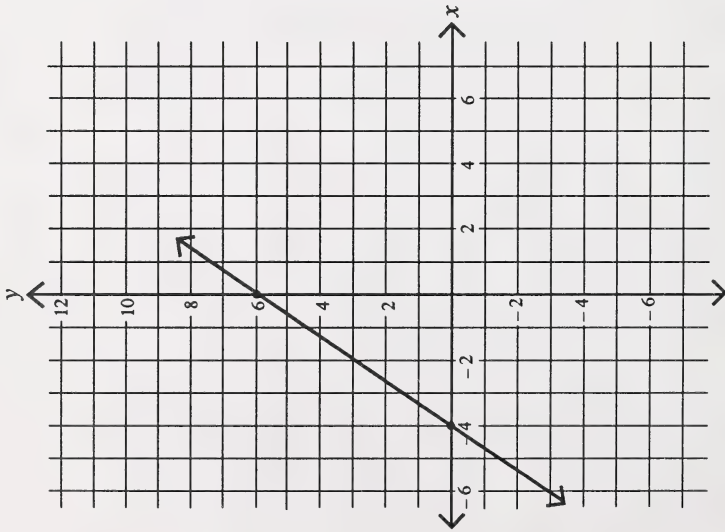
The ordered pairs are $(-3, 3)$, $(-2, 0)$, $(-1, -3)$, $(0, -6)$, and $(1, -9)$.



- b. The x -intercept is -2 .
The y -intercept is -6 .
- c. The line cuts the x -axis at $(-2, 0)$.
The line cuts the y -axis at $(0, -6)$.

8. a. The x -intercept of the line is -4 .
The y -intercept of the line is 6 .
Therefore, the line intersects the x -axis at $(-4, 0)$ and the y -axis at $(0, 6)$.

Note that since the line rises to the right, the slope is positive.



- b. The slope of the line is as follows:

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{6}{4}$$

$$= \frac{3}{2}$$

Activity 2

Write and graph the equation of a line in the form $y = mx + b$.

Given Form	$y = mx + b$	m	b	x -intercept
1. $2x + y - 7 = 0$	$y = -2x + 7$	-2	7	$\frac{7}{2} = 3.5$
2. $3y = 2x - 9$	$y = \frac{2}{3}x - 3$	$\frac{2}{3}$	-3	$\frac{9}{2} = 4.5$
3. $4x + 5y = 20$	$y = -\frac{4}{5}x + 4$	$-\frac{4}{5}$	4	5
4. $x - 3y = -9$	$y = \frac{1}{3}x + 3$	$\frac{1}{3}$	3	-9
5. $y - 4 = 0$	$y = 0x + 4$	0	4	none

For question 2, the x -intercept is found by letting $y = 0$.

$$3y = 2x - 9$$

$$3(0) = 2x - 9$$

$$0 = 2x - 9$$

$$0 + 9 = 2x - 9 + 9$$

$$9 = 2x$$

$$\frac{2x}{2} = \frac{9}{2}$$

$$x = 4\frac{1}{2} \text{ or } 4.5$$

For question 3, the x -intercept is found by letting $y = 0$.

$$4x + 5y = 20$$

$$4x + 5(0) = 20$$

$$4x + 0 = 20$$

$$4x = 20$$

$$\frac{4x}{4} = \frac{20}{4}$$

$$x = 5$$

For question 4, the x -intercept is found by letting $y = 0$.

$$x - 3y = -9$$

$$x - 3(0) = -9$$

$$x - 0 = -9$$

$$x = -9$$

For question 1, the x -intercept is found by letting $y = 0$.

$$2x + y - 7 = 0$$

$$\text{or } y = -2x + 7$$

$$2x + 0 - 7 = 0$$

$$0 = -2x + 7$$

$$2x - 7 = 0$$

$$0 - 7 = -2x + 7 - 7$$

$$2x - 7 + 7 = 0 + 7$$

$$-7 = -2x$$

$$2x = 7$$

$$\frac{-2x}{-2} = \frac{-7}{-2}$$

$$\frac{2x}{2} = \frac{7}{2}$$

$$\frac{-7}{-2} = \frac{-7}{-2}$$

$$x = \frac{7}{2} \text{ or } 3.5$$

$$x = 3\frac{1}{2} \text{ or } 3.5$$

For question 5, the x-intercept is found by letting $y = 0$.

$$y - 4 = 0$$

$$0 - 4 = 0$$

$$-4 = 0$$

This is not true. Therefore, there is no x-intercept for $y - 4 = 0$.

6. a.

$$y = 6 - 2x$$

$$y - y = 6 - 2x - y$$

$$0 = -2x - y + 6$$

$$-2x - y + 6 = 0$$

$$2x + y - 6 = 0$$

Note that $-2x - y + 6 = 0$ was changed to $2x + y - 6 = 0$ by multiplying every term of the equation by -1 .

b.

$$2y + 3x = \frac{1}{2}$$

$$2(2y + 3x) = 2\left(\frac{1}{2}\right)$$

$$4y + 6x = 1$$

$$4y + 6x - 1 = 1 - 1$$

$$4y + 6x - 1 = 0$$

$$6x + 4y - 1 = 0$$

c. $\frac{x-2}{3} = y$

$$\frac{3(x-2)}{3} = 3y$$

$$x-2 = 3y$$

$$x-3y-2=0$$

d.

$$y = mx + b$$

$$y - y = mx - y + b$$

$$0 = mx - y + b$$

$$mx - y + b = 0$$

e.

$$\frac{x}{2} - \frac{y}{3} + 1 = 0$$

$$6\left(\frac{x}{2} - \frac{y}{3} + 1\right) = 6(0)$$

$$\frac{6x}{2} - \frac{6y}{3} + 6 = 0$$

$$3x - 2y + 6 = 0$$

7. $y = 3x + 2 \leftarrow$ y-intercept

↑
slope

$$\text{slope} = m = 3$$

$$\text{y-intercept} = 2$$

The y-intercept would be the point $(0, 2)$.

$$m = \frac{\text{rise}}{\text{run}}$$

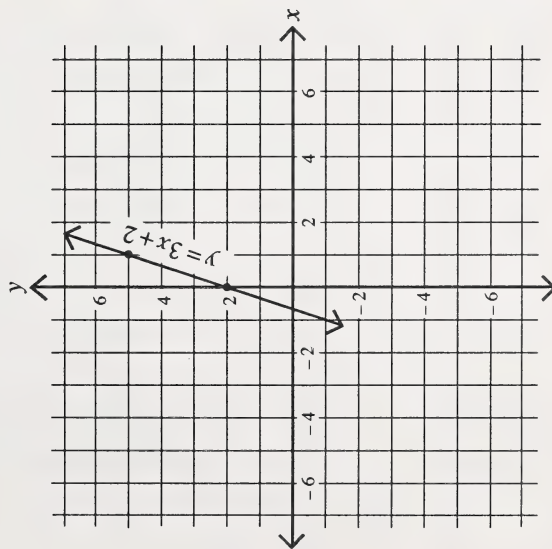
$$= 3$$

$$= \frac{3}{1}$$

$$\text{rise} = 3$$

$$\text{run} = 1$$

Start at the point $(0, 2)$ which is the y-intercept. Go one to the right and the three up to find the second point.



Extra Help

$$1. \quad a. \quad 2x - 3y - 7 = 0$$

$$-3y = -2x + 7$$

$$\frac{-3y}{-3} = \frac{-2x}{-3} + \frac{7}{-3}$$

$$y = \frac{2}{3}x - \frac{7}{3}$$

Note that $\frac{2}{3}$ is the same as $-\frac{-2}{3}$ or $-\frac{7}{3}$.
The slope is $\frac{2}{3}$.

$$b. \quad \text{The y-intercept is } -\frac{7}{3}.$$

$$2. \quad \text{For the x-intercept, let } y = 0.$$

$$2x + 3y - 12 = 0$$

$$2x + 3(0) - 12 = 0$$

$$2x - 12 = 0$$

$$2x = 12$$

$$\frac{2x}{2} = \frac{12}{2}$$

$$x = 6$$

The x-intercept is 6.

For the y-intercept, let $x = 0$.

$$2x + 3y - 12 = 0$$

$$2(0) + 3y - 12 = 0$$

$$3y = 12$$

$$\frac{3y}{3} = \frac{12}{3}$$

$$y = 4$$

The y-intercept is 4.

3. Let $y = 0$ to find the x-intercept.

$$4x - 3y - 12 = 0$$

$$4x - 3(0) - 12 = 0$$

$$4x - 12 = 0$$

$$4x = 12$$

$$\frac{4x}{4} = \frac{12}{4}$$

$$x = 3$$

The x-intercept is 3.

Let $x = 0$ to find the y-intercept.

$$4x - 3y - 12 = 0$$

$$4(0) - 3y - 12 = 0$$

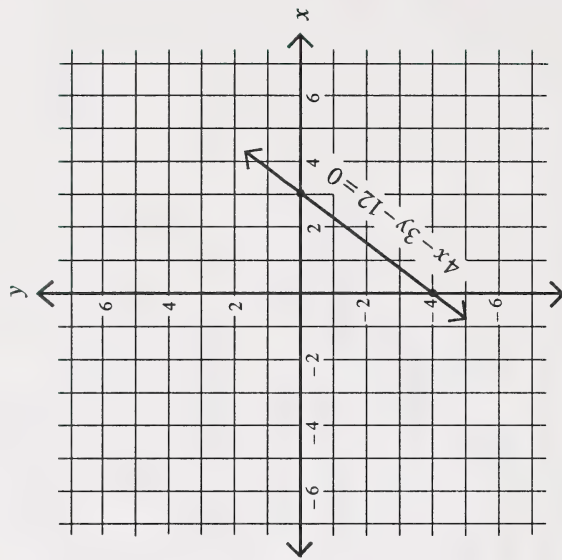
$$-3y - 12 = 0$$

$$-3y = 12$$

$$\frac{-3y}{-3} = \frac{12}{-3}$$

$$y = -4$$

The y-intercept is -4 .



4. $2x - 6 = 0$

$2x = 6$

$\frac{2x}{2} = \frac{6}{2}$

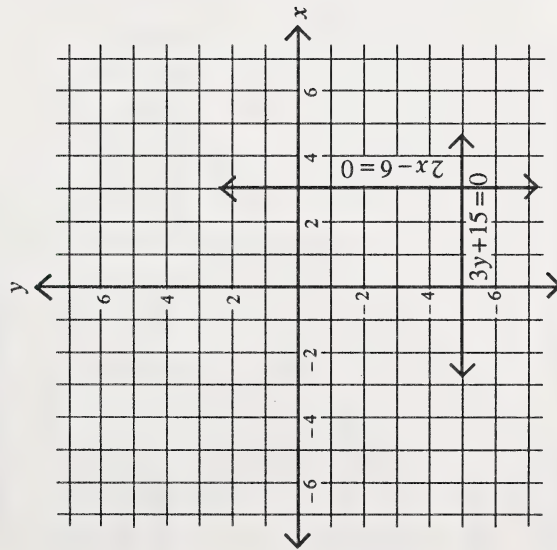
$x = 3$

$3y + 15 = 0$

$3y = -15$

$\frac{3y}{3} = \frac{-15}{3}$

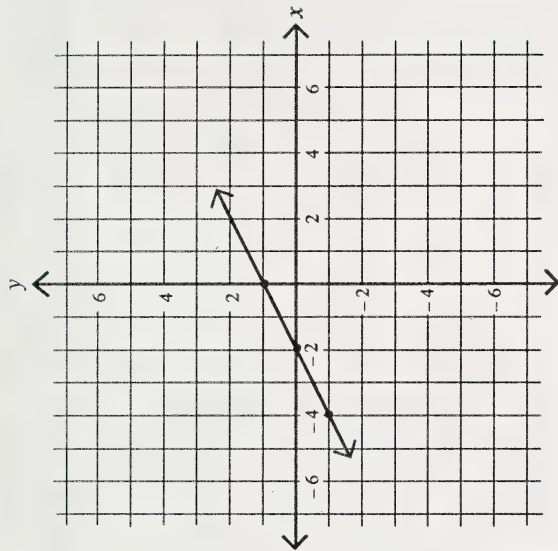
$y = -5$



For every point on $2x - 6 = 0$, the first coordinate is 3.

For every point on $3y + 15 = 0$, the second coordinate is -5 .

5.



Since the line passes through $(-2, 0)$ and $(0, 1)$, the slope

$m = \frac{\text{rise}}{\text{run}} = \frac{1}{2}$. The slope is always positive when the line rises to the right.

From the graph, b can be found to be 1 since the y -intercept is found at $(0, 1)$. Now you can use the slope y -intercept form of a linear equation to find the equation.

$$y = mx + b$$

$$y = \frac{1}{2}x + 1$$

$$2(y) = 2\left(\frac{1}{2}x\right) + 2(1)$$

$$2y = x + 2$$

$$x - 2y + 2 = 0$$

The equation of the line on the coordinate plane is

$$x - 2y + 2 = 0.$$

Extensions

1. $3x - 2y = 6$

$$-2y = -3x + 6$$

$$y = \frac{-3}{-2}x + \frac{6}{-2}$$

$$y = \frac{3}{2}x - 3$$

Note that $y = mx + b$, where m is the slope and b is the y -intercept.

$$\therefore m = \frac{3}{2}$$

a. When lines are parallel, $m_1 = m_2$.

The slope of $3x - 2y = 6$ is $\frac{3}{2}$. A line parallel to

$3x - 2y = 6$ will also have a slope of $\frac{3}{2}$.

b. When lines are perpendicular, $m_1 m_2 = -1$.

$$\text{Let } m_1 = \frac{3}{2}.$$

$$\therefore \left(\frac{3}{2}\right)m_2 = -1$$

$$m_2 = -1 + \frac{3}{2}$$

$$= -1 \times \frac{2}{3}$$

$$= -\frac{2}{3}$$

The slope of a line perpendicular to $3x - 2y = 6$ is $-\frac{2}{3}$.

2. The two points given are $A(0, 1)$ and $B(4, 4)$.

Let the slope of a line through A and B be m .

$$\therefore m = \frac{\text{rise}}{\text{run}}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 1}{4 - 0}$$

$$= \frac{3}{4}$$

Since $y = 1$ when $x = 0$, the line intersects the y -axis at $(0, 1)$ and the y -intercept is 1.

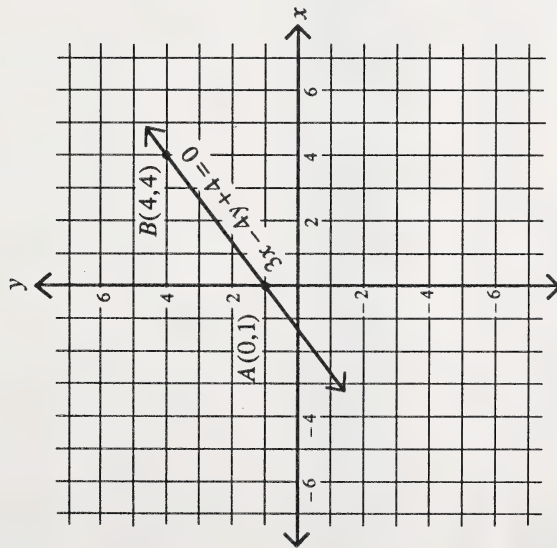
$$y = mx + b$$

$$y = \frac{3}{4}x + 1$$

$$4(y) = 4\left(\frac{3}{4}x\right) + 4(1)$$

$$4y = 3x + 4$$

$$3x - 4y + 4 = 0$$



3. The equation for side AC is as follows:

$$x - 3y + 8 = 0$$

$$3y = x + 8$$

$$\frac{3y}{3} = \frac{x + 8}{3}$$

$$y = \frac{1}{3}x + \frac{8}{3}$$

Thus, the slope of $\overline{AC}$ is $\frac{1}{3}$.

The equation for side BC is as follows:

$$3x + y - 16 = 0$$

$$y = -3x - 16$$

Thus, the slope of $\overline{BC}$ is -3 .

The equation for side AB is as follows:

$$x + 2y - 7 = 0$$

$$2y = -x + 7$$

$$\frac{2y}{2} = \frac{-x + 7}{2}$$

$$y = -\frac{1}{2}x + \frac{7}{2}$$

Thus, the slope of $\overline{AB}$ is $-\frac{1}{2}$.

The slope of $\overline{AC}$ is $\frac{1}{3}$ and the slope of $\overline{BC}$ is -3 . The product of the two slopes is -1 so a right angle exists at C . Since one angle of triangle ABC is a right angle, it is a right triangle.

Remember these points.

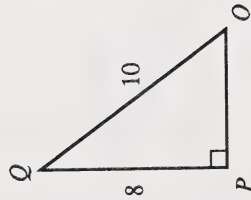
- Parallel lines have the same slope.
- The slopes of perpendicular lines are negative reciprocals.
- Note that $-x$ is the same as $-1x$.



Exploring Topic 4

Activity 1

Use the Pythagorean theorem and the distance formula to find the distance between two points.



Use the Pythagorean theorem.

$$(OQ)^2 = (QP)^2 + (OP)^2$$

$$10^2 = 8^2 + (OP)^2$$

$$(OP)^2 = 10^2 - 8^2$$

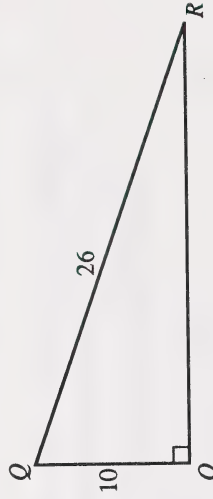
$$(OP)^2 = 100 - 64$$

$$(OP)^2 = 36$$

$$OP = \sqrt{36}$$

$$= 6$$

Side OP is six units in length.



$$(QR)^2 = (OQ)^2 + (OR)^2$$

$$26^2 = 10^2 + (OR)^2$$

$$(OR)^2 = 26^2 - 10^2$$

$$(OR)^2 = 676 - 100$$

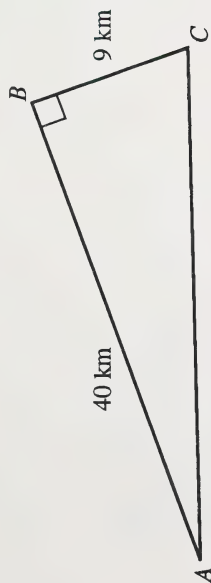
$$(OR)^2 = 576$$

$$OR = \sqrt{576}$$

$$= 24$$

Side OR is 24 units in length.

2. Since the locations of A, B, and C form a right triangle, you can write the following:



$$(AC)^2 = (AB)^2 + (BC)^2$$

$$(AC)^2 = 40^2 + 9^2$$

$$(AC)^2 = 1600 + 81$$

$$(AC)^2 = 1681$$

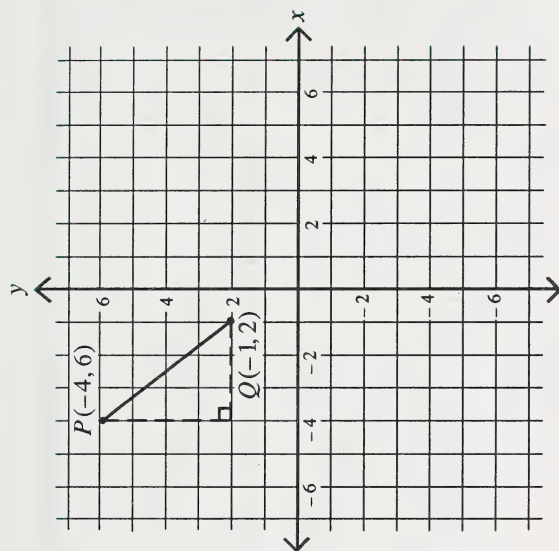
$$AC = \sqrt{1681}$$

$$= 41$$

$$= 41 \text{ km}$$

Since the courier travelled through Town B, the courier went $(40+9)$ km going to Town C. On returning, 41 km were covered.

The courier saved $49 \text{ km} - 41 \text{ km}$ or 8 km by taking the shortcut route when returning to Town A from Town C.



From the diagram, you get the following:

$$\text{rise} = y_2 - y_1$$

$$= 6 - 2$$

$$= 4$$

$$\text{run} = x_2 - x_1$$

$$= -4 - (-1)$$

$$= -4 + 1$$

$$= -3$$

$$(PQ)^2 = 4^2 + (-3)^2$$

$$(PQ)^2 = 16 + 9$$

$$(PQ)^2 = 25$$

$$PQ = \sqrt{25}$$

$$= 5$$

Segment PQ is five units in length.

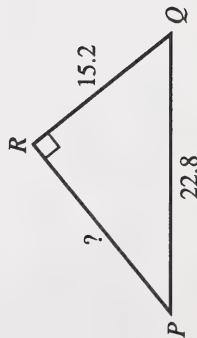
4. Since PQR is a right triangle, the Pythagorean theorem holds.

$$\therefore (PQ)^2 = (QR)^2 + (PR)^2$$

This can be rewritten as follows:

$$(PR)^2 = (PQ)^2 - (QR)^2$$

$$(PR)^2 = (22.8)^2 - (15.2)^2$$



Thus, side $PR \approx 16.994$ 116 63 or approximately 17 m.

5. You are told that $A(-3, 8)$ and $B(5, 7)$ are the two points that you use to find the required distance.

The distance formula states that $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Use your calculator as follows:

Enter	Display
22.8	22.8
x^2	519.84
$-$	519.84
15.2	15.2
x^2	15.2
$-$	231.04
$=$	288.8
$\sqrt{}$	16.99411663

Let $x_1 = -3$.

Let $x_2 = 5$.

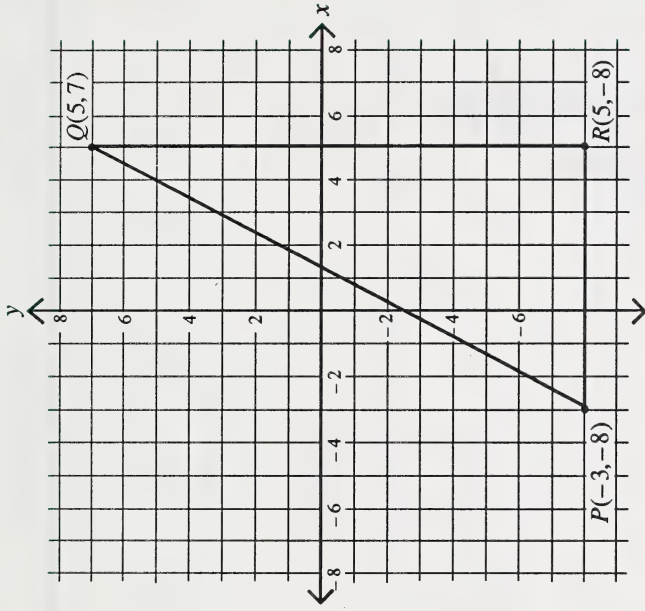
Let $y_1 = 8$.

Let $y_2 = 7$.

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{[5 - (-3)]^2 + (7 - 8)^2} \\&= \sqrt{(8^2) + (-1)^2} \\&= \sqrt{64 + 1} \\&= \sqrt{65} \\&= 8.06\end{aligned}$$

The distance is 8.06 units.

6. You could plot the points and then connect them to form a triangle.



$$\begin{aligned}PQ &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\&= \sqrt{[5 - (-3)]^2 + [7 - (-8)]^2} \\&= \sqrt{8^2 + 15^2} \\&= \sqrt{64 + 225} \\&= \sqrt{289} \\&= 17\end{aligned}$$

$$\begin{aligned}
 QR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(5 - 5)^2 + (-8 - 7)^2} \\
 &= \sqrt{0^2 + (-15)^2} \\
 &= \sqrt{0 + 225} \\
 &= \sqrt{225} \\
 &= 15
 \end{aligned}$$

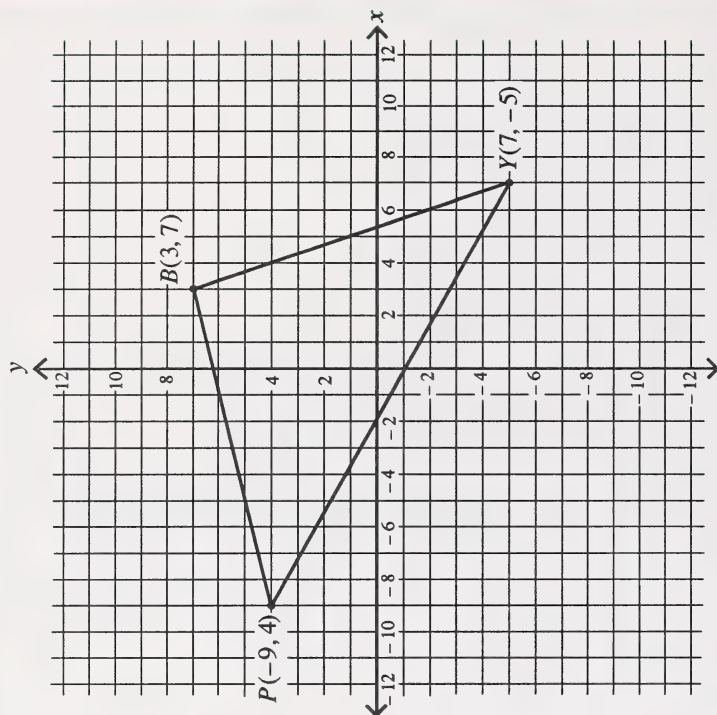
$$\begin{aligned}
 PR &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[5 - (-3)]^2 + [-8 - (-8)]^2} \\
 &= \sqrt{8^2 + (-8 + 8)^2} \\
 &= \sqrt{64 + 0} \\
 &= \sqrt{64} \\
 &= 8
 \end{aligned}$$

Because $\overline{PR}$ is horizontal, you should see quickly by counting that the run is eight units. Since $\overline{QR}$ is vertical, the spaces can be counted to get 15.

$$\begin{aligned}
 \text{Perimeter} &= PQ + QR + PR \\
 &= 17 + 15 + 8 \\
 &= 40
 \end{aligned}$$

The perimeter of $\triangle PQR$ is 40 units.

7. You could plot the points and then connect them to form a triangle.



- a. Let the distance from the boat to the yacht be BY .

$$\begin{aligned}
 BY &= \sqrt{(7-3)^2 + (-5-7)^2} \\
 &= \sqrt{(4)^2 + (-12)^2} \\
 &= \sqrt{16+144} \\
 &= \sqrt{160} \\
 &= 12.649\ 110\ 64 \\
 &= 12.65
 \end{aligned}$$

The distance from the boat to the yacht is 12.65 knots.

- b. Let the distance from the boat to the port be BP .

$$\begin{aligned}
 BP &= \sqrt{[3-(-9)]^2 + (7-4)^2} \\
 &= \sqrt{(12)^2 + (3)^2} \\
 &= \sqrt{144+9} \\
 &= \sqrt{153} \\
 &= 12.369\ 316\ 88 \\
 &= 12.37
 \end{aligned}$$

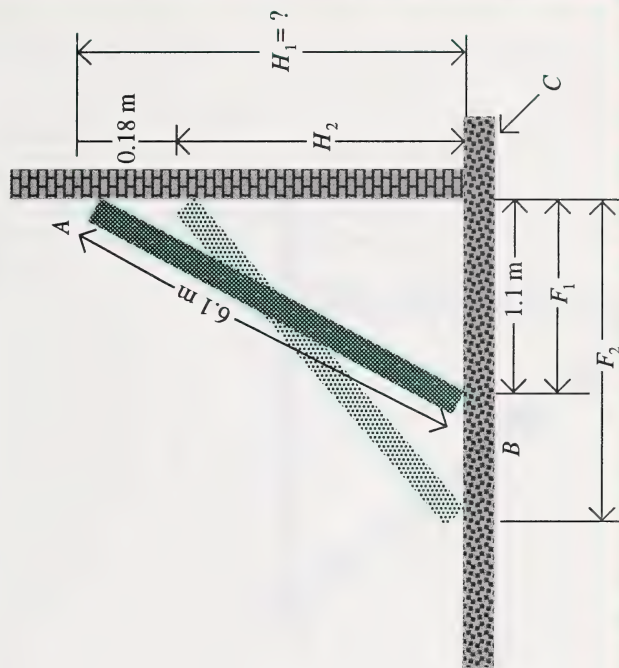
The distance from the boat to the port is 12.37 knots.

- c. Let the distance from the port to the yacht be PY .

$$\begin{aligned}
 PY &= \sqrt{[7-(-9)]^2 + (-5-4)^2} \\
 &= \sqrt{(16)^2 + (-9)^2} \\
 &= \sqrt{256+81} \\
 &= \sqrt{337} \\
 &= 18.357\ 559\ 75 \\
 &= 18.36
 \end{aligned}$$

The distance from the port to the yacht is 18.36 knots.

8.



- a. Let the bottom of the wall be C. By the Pythagorean theorem, find the height that the pole reaches up the wall before it slips down.

$$c^2 = a^2 + b^2$$

$$(6.1)^2 = (1.1)^2 + (H_1)^2$$

$$(H_1)^2 = (6.1)^2 - (1.1)^2$$

$$(H_1)^2 = 37.21 - 1.21$$

$$(H_1)^2 = 36$$

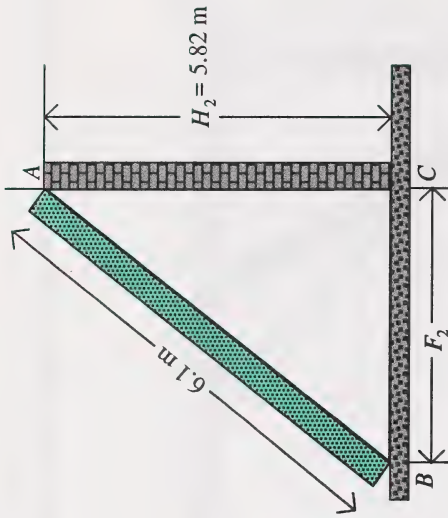
$$\sqrt{(H_1)^2} = \sqrt{36}$$

$$H_1 = 6$$

The pole reaches 6.0 m up the wall before it slips.

- b. $H_2 = 6.0 - 0.18$
 $= 5.82 \text{ m}$

The length of the pole is unchanged. Use the Pythagorean theorem again.



$$c^2 = a^2 + b^2$$

$$(6.1)^2 = (5.82)^2 + (F_2)^2$$

$$(F_2)^2 = (6.1)^2 - (5.82)^2$$

$$(F_2)^2 = 37.21 - 33.8724$$

$$(F_2)^2 = 3.3376$$

$$\sqrt{(F_2)^2} = \sqrt{3.3376}$$

$$F_2 \doteq 1.826909959$$

$$\doteq 1.83$$

The foot of the pole will be about 1.83 m from the base of the wall after it slips 0.18 m down the wall.

Note that although the pole slipped only 0.18 m down the wall, it moved $1.83 - 1.1 = 0.73$ m along the ground.

Activity 2

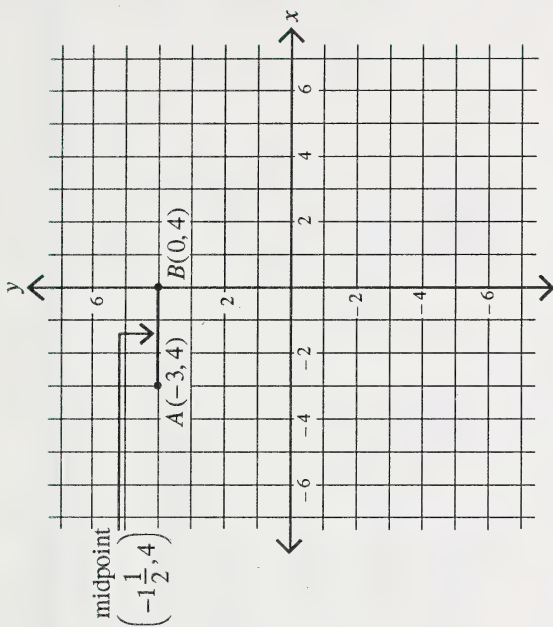
Use the formula $x = \frac{x_1 + x_2}{2}$, $y = \frac{y_1 + y_2}{2}$ to find the midpoint of a line segment.

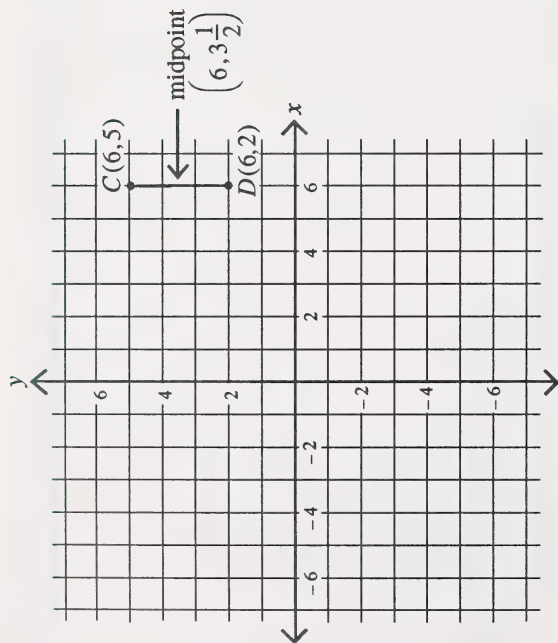
$$\begin{aligned}
 1. \quad x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\
 &= \frac{-3 + 0}{2} & &= \frac{4 + 4}{2} \\
 &= \frac{-3}{2} & &= \frac{8}{2} \\
 &= -1\frac{1}{2} & &= 4
 \end{aligned}$$

The midpoint of $\overline{AB}$ is $(-1\frac{1}{2}, 4)$.

$$\begin{aligned}
 2. \quad x &= \frac{x_1 + x_2}{2} & y &= \frac{y_1 + y_2}{2} \\
 &= \frac{6 + 6}{2} & &= \frac{5 + 2}{2} \\
 &= \frac{12}{2} & &= \frac{7}{2} \\
 &= 6 & &= 3\frac{1}{2}
 \end{aligned}$$

The midpoint of $\overline{CD}$ is $(6, 3\frac{1}{2})$.

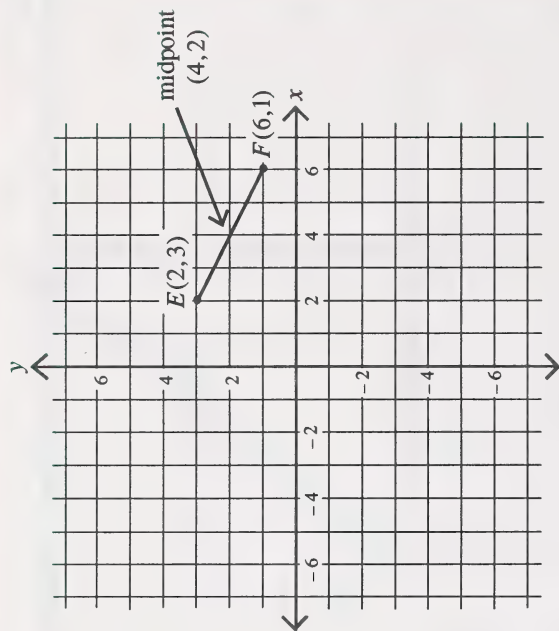




$$\begin{aligned} 3. \quad x &= \frac{x_1 + x_2}{2} \\ &= \frac{2 + 6}{2} \\ &= \frac{8}{2} \\ &= 4 \end{aligned}$$

The midpoint of $\overline{EF}$ is $(4, 2)$.

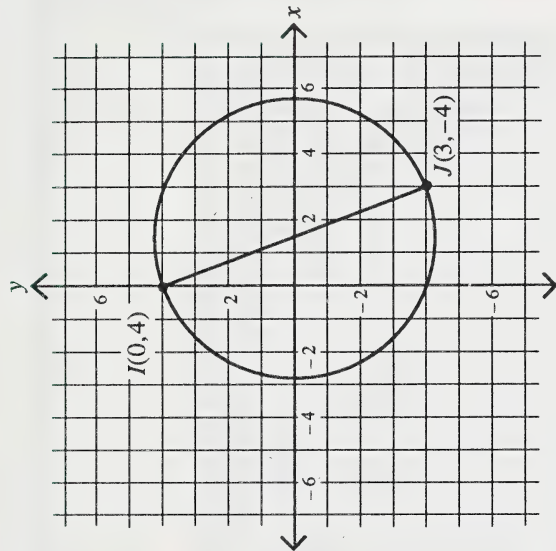
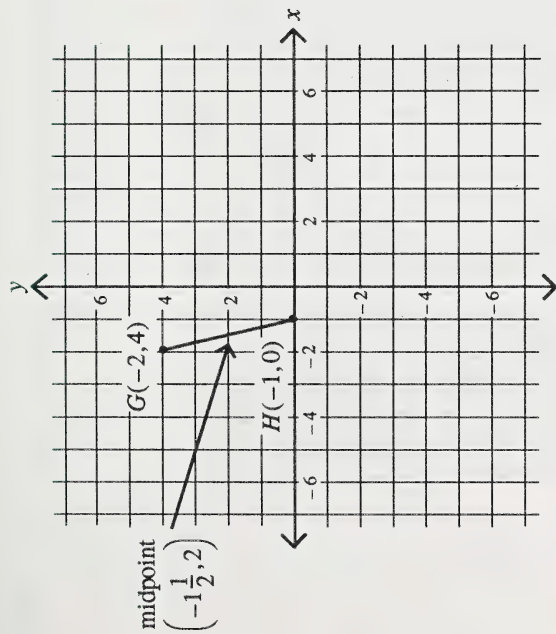
$$\begin{aligned} y &= \frac{y_1 + y_2}{2} \\ &= \frac{3 + 1}{2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$



$$\begin{aligned} y &= \frac{y_1 + y_2}{2} \\ &= \frac{4 + 0}{2} \\ &= \frac{4}{2} \\ &= 2 \end{aligned}$$

$$\begin{aligned} 4. \quad x &= \frac{x_1 + x_2}{2} \\ &= \frac{-2 + (-1)}{2} \\ &= \frac{-3}{2} \\ &= -1\frac{1}{2} \end{aligned}$$

The midpoint of $\overline{GH}$ is $(-1\frac{1}{2}, 2)$.



The centre of the circle is the point $(1\frac{1}{2}, 0)$.

Extra Help

- The area of the square drawn on the hypotenuse is $c^2 = 5^2 = \underline{25}$ square units.
 - The areas of the squares drawn on the other two sides are $a^2 = \underline{4}^2 = \underline{16}$ square units and $b^2 = \underline{3}^2 = \underline{9}$ square units.

$$\begin{aligned}
 5. \quad x &= \frac{x_1 + x_2}{2} \\
 &= \frac{0 + 3}{2} \\
 &= \frac{3}{2} \\
 &= 1\frac{1}{2}
 \end{aligned}$$

The midpoint of $\overline{IJ}$ is $(1\frac{1}{2}, 0)$. Therefore, the centre of the circle is $(1\frac{1}{2}, 0)$.

c. $c^2 = a^2 + b^2$

$$5^2 = \frac{4^2}{16} + \frac{3^2}{9}$$

$$25 = \frac{16}{16} + \frac{9}{9}$$

$$25 = \frac{25}{1}$$

2.

Triple (a, b, c)	a^2	b^2	$a^2 + b^2$	c^2	Does $c^2 = a^2 + b^2$?
(6, 8, 10)	$6^2 = 36$	$8^2 = 64$	$36 + 64 = 100$	$10^2 = 100$	Yes
(4, 5, 6)	$4^2 = 16$	$5^2 = 25$	$16 + 25 = 41$	$6^2 = 36$	No
(12, 15, 18)	$12^2 = 144$	$15^2 = 225$	$144 + 225 = 369$	$18^2 = 324$	No
(9, 12, 15)	$9^2 = 81$	$12^2 = 144$	$81 + 144 = 225$	$15^2 = 225$	Yes
(5, 8, 9)	$5^2 = 25$	$8^2 = 64$	$25 + 64 = 89$	$9^2 = 81$	No
(10, 24, 26)	$10^2 = 100$	$24^2 = 576$	$100 + 576 = 676$	$26^2 = 676$	Yes
(1, 2, $\sqrt{3}$)	$1^2 = 1$	$2^2 = 4$	$1 + 4 = 5$	$(\sqrt{3})^2 = 3$	No

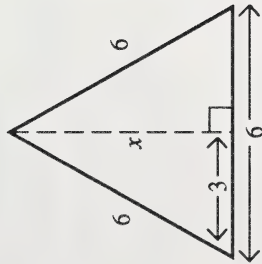
Note the following:

For (6, 8, 10), if each side were divided by 2, you would have a 3 - 4 - 5 triangle.

For (9, 12, 15), if each side were divided by 3, you would have a 3 - 4 - 5 triangle.

For (10, 24, 26), if each side were divided by 2, you would have a 5 - 12 - 13 triangle.

3. a.



$$6^2 = x^2 + 3^2$$

$$x^2 = 6^2 - 3^2$$

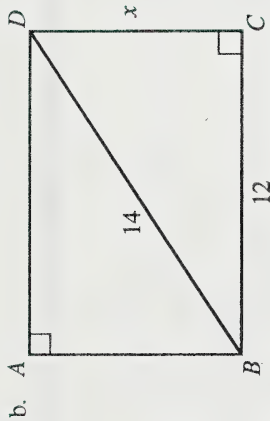
$$x^2 = 36 - 9$$

$$x^2 = 27$$

$$\sqrt{x^2} = \sqrt{27}$$

$$x \doteq 5.196152423$$

$$\doteq 5.2 \quad (\text{rounded to one decimal place})$$



$$c^2 = a^2 + b^2$$

$$(BD)^2 = (BC)^2 + (DC)^2$$

$$14^2 = 12^2 + x^2$$

$$x^2 = 14^2 - 12^2$$

$$x^2 = 196 - 144$$

$$x^2 = 52$$

$$\sqrt{x^2} = \sqrt{52}$$

$$x \doteq 7.211102551$$

$$\doteq 7.2 \quad (\text{rounded to one decimal place})$$

Extensions

1. $A = \left(-4\frac{1}{3}, -6\frac{1}{2}\right)$ $B\left(5\frac{1}{2}, -1\frac{3}{4}\right)$

$$\begin{aligned}
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{\left[5\frac{1}{2} - \left(-4\frac{1}{3}\right)\right]^2 + \left[-1\frac{3}{4} - \left(-6\frac{1}{2}\right)\right]^2} \\
&= \sqrt{\left(5\frac{1}{2} + 4\frac{1}{3}\right)^2 + \left(-1\frac{3}{4} + 6\frac{1}{2}\right)^2} \\
&= \sqrt{\left(5\frac{3}{6} + 4\frac{2}{6}\right)^2 + \left(-1\frac{3}{4} + 6\frac{2}{4}\right)^2} \\
&= \sqrt{\left(9\frac{5}{6}\right)^2 + \left(6\frac{2}{4} - 1\frac{3}{4}\right)^2} \\
&= \sqrt{\left(9\frac{5}{6}\right)^2 + \left(5\frac{6}{4} - 1\frac{3}{4}\right)^2} \\
&= \sqrt{\left(9\frac{5}{6}\right)^2 + \left(4\frac{3}{4}\right)^2} \\
&= \sqrt{\left(\frac{59}{6}\right)^2 + \left(\frac{19}{4}\right)^2} \\
&\doteq \sqrt{(9.8\bar{3})^2 + (4.75)^2} \\
&\doteq \sqrt{96.694\,444\,44 + 22.5625} \\
&\doteq \sqrt{119.257\,444\,44} \\
&\doteq 10.920\,505\,69 \\
&\doteq 10.92
\end{aligned}$$

The distance between points A and B is approximately 10.92 units.

$$\begin{aligned}
2. \quad A &= \left(6\frac{1}{5}, 3\frac{1}{4}\right) & B &= \left(-1\frac{1}{8}, -6\frac{5}{9}\right) \\
d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
&= \sqrt{\left(-1\frac{1}{8} - 6\frac{1}{5}\right)^2 + \left(-6\frac{5}{9} - 3\frac{1}{4}\right)^2} \\
&= \sqrt{\left(-1\frac{5}{40} - 6\frac{8}{40}\right)^2 + \left(-6\frac{20}{36} - 3\frac{9}{36}\right)^2} \\
&= \sqrt{\left(-7\frac{13}{40}\right)^2 + \left(-9\frac{29}{36}\right)^2} \\
&= \sqrt{\left(\frac{-293}{40}\right)^2 + \left(\frac{-353}{36}\right)^2} \\
&= \sqrt{(-7.325)^2 + (-9.80\bar{5})^2} \\
&\doteq \sqrt{53.655\,625 + 96.148\,919\,75} \\
&\doteq \sqrt{149.804\,544\,8} \\
&\doteq 12.239\,466\,69 \\
&\doteq 12.24
\end{aligned}$$

The distance between points A and B is approximately 12.24 units.

3. $A(-4.3, -6.5)$ and $B(5.5, -1.75)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{[5.5 - (-4.3)]^2 + [-1.75 - (-6.5)]^2} \\
 &= \sqrt{(9.8)^2 + (4.75)^2} \\
 &= \sqrt{96.04 + 22.5625} \\
 &= \sqrt{118.6025} \\
 &= 10.89047749 \\
 &= 10.89
 \end{aligned}$$

The distance between points A and B is 10.89 units. Compare this answer with question 1.

Because -4.3 is almost $-4\frac{1}{3}$ or $-4\frac{1}{3}$, -6.5 is equal to $-6\frac{1}{2}$, 5.5 is equal to $5\frac{1}{2}$, and -1.75 is equal to $-1\frac{3}{4}$, you would expect the answers to be very close and they are. One answer is about 10.92 units and the other is about 10.89 units.

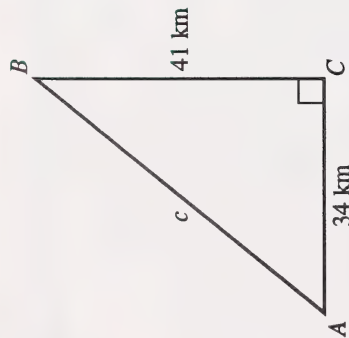
4. $A(6.2, 3.25)$ and $B(-1.125, -6.56)$

$$\begin{aligned}
 d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
 &= \sqrt{(-1.125 - 6.2)^2 + (-6.56 - 3.25)^2} \\
 &= \sqrt{(-7.325)^2 + (-9.81)^2} \\
 &= \sqrt{53.655625 + 96.2361} \\
 &= \sqrt{149.891725} \\
 &= 12.24302761 \\
 &= 12.24
 \end{aligned}$$

The distance between points A and B is about 12.24 units. Compare this answer with question 2.

Because 6.2 is equal to $6\frac{1}{5}$, 3.25 is equal to $3\frac{1}{4}$, -1.125 is equal to $-1\frac{1}{8}$, and -6.56 is almost $6\frac{5}{9}$ or $6\frac{5}{9}$, you would expect the answers to be very close. In fact they are the same. Both answers are about 12.24 units.

5.



Use the Pythagorean theorem to find the distance Joe walks.

$$c^2 = a^2 + b^2$$

$$c^2 = 41^2 + 34^2$$

$$c^2 = 1681 + 1156$$

$$c^2 = 2837$$

$$c = \sqrt{2837}$$

$$\doteq 53.263\,495\,94$$

$$\doteq 53.3 \text{ km} \quad (\text{rounded to one decimal place})$$

Add AC and CB to find the distance that Ernie runs.

$$AC + CB = 34 \text{ km} + 41 \text{ km}$$

$$= 75 \text{ km}$$

Use the formula $t = \frac{d}{r}$ to find the time it takes Joe to walk 53.3 km at 5 km/h.

$$t = \frac{d}{r}$$

$$= \frac{53.3}{5}$$

$$= 10.66$$

$$= 10.7 \quad (\text{rounded to one decimal place})$$

It takes Joe about 10.7 h to walk from Point A to Point B.

Use the formula $t = \frac{d}{r}$ to find the time it takes Ernie to run 75 km at 7.5 km/h.

$$t = \frac{d}{r}$$

$$= \frac{75}{7.5}$$

$$= 10 \text{ h}$$

It takes Ernie 10 h to run from Point A to Point C and then on to Point B.

It would take Joe $10.7 - 10 = 0.7$ h longer to get to Point B than it would take Ernie. Ernie would be at Point B first and would be waiting for about $0.7 \times 60 = 42$ minutes for Joe to arrive.



Mathematics 13

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